

Module-07

Introduction to Involute Gear Tooth Correction

Lecture-33

Involute Spur Gear Tooth Correction : Part-I

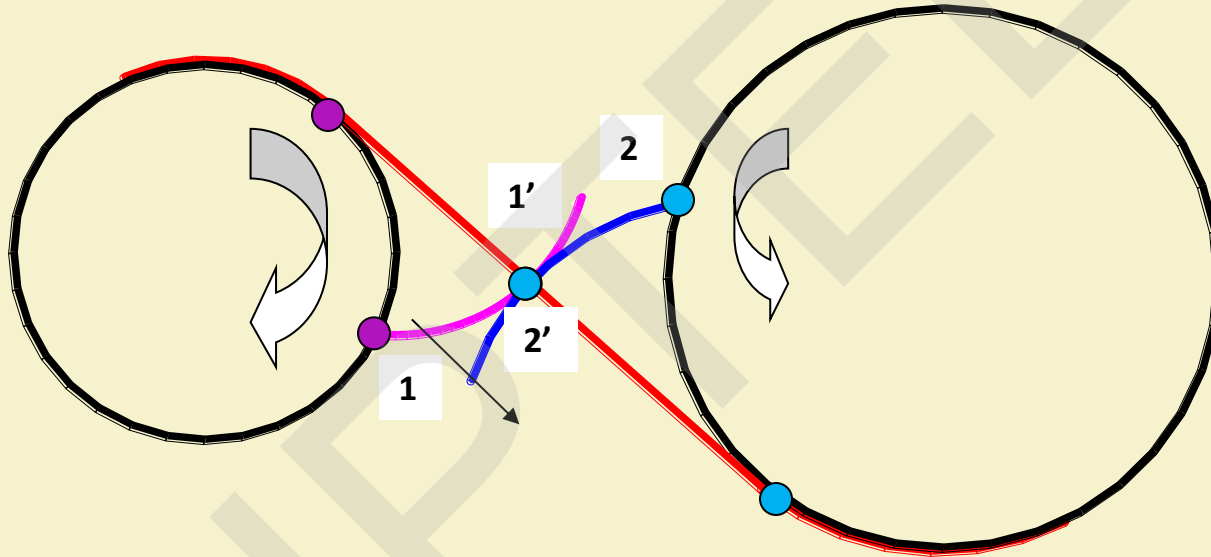


Outline of the Lecture

- **Concept of Involute Tooth (Recapitulation)**
 - **Generation of Involute Tooth**
 - **Basic Rack and Rack Cutter**
 - **Concept of Gear Tooth Correction**
 - **Why Gear Tooth Correction**
 - **Plus & Minus Correction**
 - **Amount of Correction (Profile Shift)**

Concept of Toothed Gear and Tooth Profile : Involute profile

Recapitulation



A cross belt drive and it's replacement by conjugate profiles.

Concept of Involute tooth .



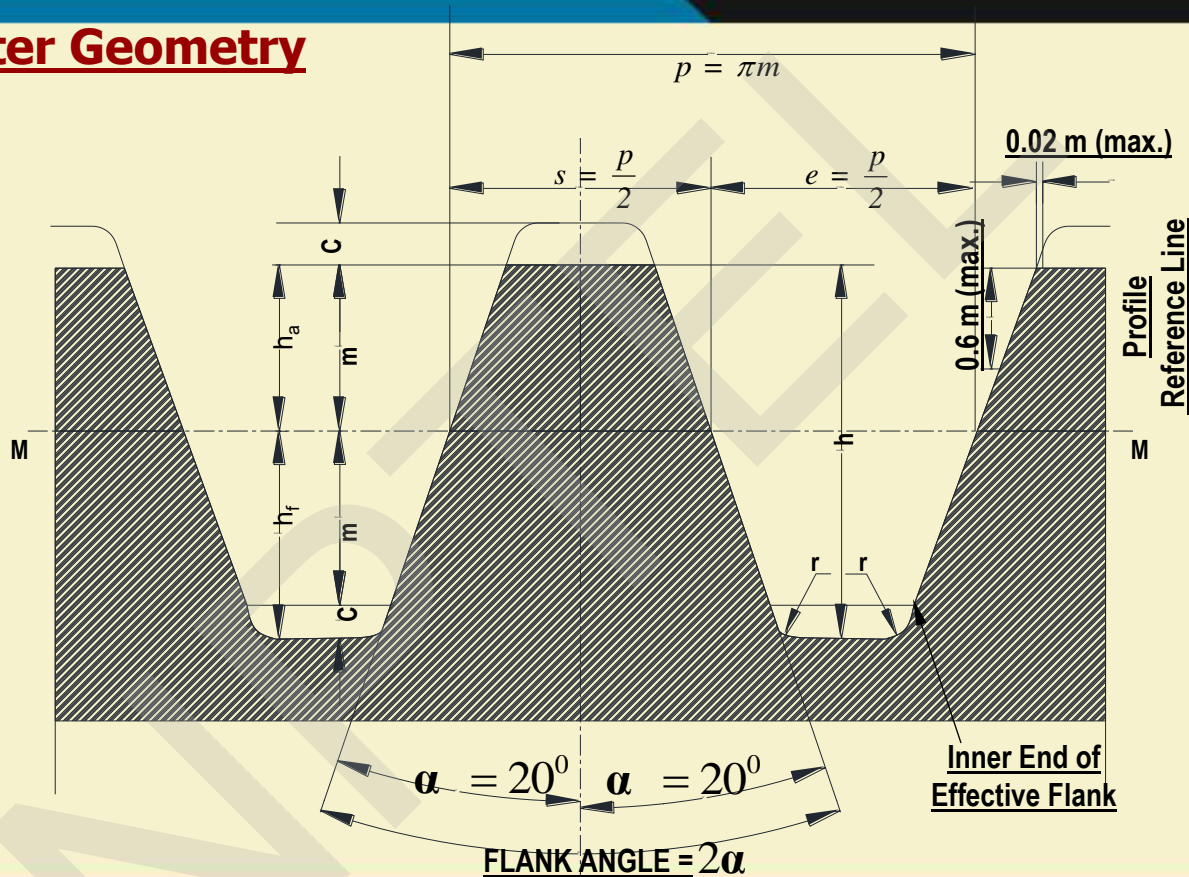
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Rack (Basic) Cutter Geometry



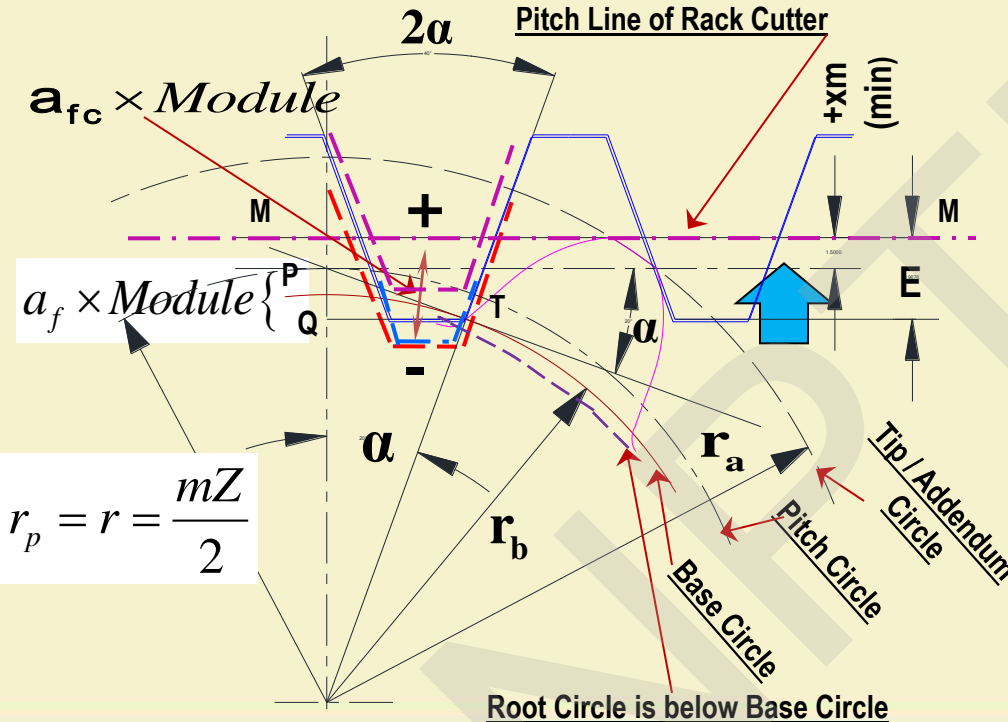
Minimum Number of Teeth in a Gear, Interference and Undercut, & Concept of Gear Tooth Correction :

Minimum (Critical) Number of Teeth:

$$Z_c = \frac{2a_f}{\sin^2 \alpha}$$

Where, $a_f \times \text{Module}$ (a_f being a factor) is the addendum height of the tooth.

and, $a_{fc} \times \text{Module}$ (a_{fc} being the cutter addendum factor) is the addendum height (active) of the rack cutter.



Introduction to Involute Spur Gear Tooth Correction :

Why Gear Tooth Correction ?

- **The tooth correction is primarily done to avoid undercut at the tooth root and dedendum region for gears with less than critical number of teeth.**
- **It is also done to modify mating gears' tooth shapes so that they become more or less of equal strength.**
- **Modification of centre distance, mainly to make the centre distance a good square value (close to original CD of uncorrected gears), is another purpose.**
- **It also helps in reduction in dynamics.**

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Two types of gear tooth corrections are done in gear cutting-

- (i) Addendum modification, and**
- (ii) Tooth profile correction.**

As already discussed, these are done to avoid interference and undercut, increase the load carrying capacity of a tooth as well as to reduce the dynamics and noise.

Addendum Modification

In gear technology such correction is termed as “profile displacement”, “profile shift”, etc.

In fact, by not allowing the cutter to reach it's ideal feed limit or allowing it to higher depth than the ideal feed limit such corrections are done.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Addendum Modification

In gear technology such correction is termed as “profile displacement”, “profile shift”, etc.

In fact, by not allowing the cutter to reach its ideal feed limit or allowing it to higher depth than the ideal feed limit such corrections are done.

The tooth height is selected accordingly.

Geometrically keeping the base drum unaltered parallel shift of the involute profile is possible.

In first case it is “positive correction” i.e. “positive addendum modification” which results in a ‘greater tooth root width’.

Obviously this increases strength in bending.

In later case root width reduces resulting in lowering the bending strength. However, rate of such changes in strength with profile shifts reduces with the increase in number of teeth.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Addendum Modification (Contd....)

Such a correction is sometime called as 'profile modification'.

However, correction or modification of the geometric shape of the profile is put into a separate category which is termed as '*tooth profile correction*'.

In a mating gear pair such a correction can be done without changing or with a minor change in centre distance (to make the centre distance a roundup figure which is usually multiple of *5 mm* in metric system) the centre distance.

It is sometimes called as '+' '-' (plus-minus) correction. '+' correction is given to the pinion and '-' correction is given to the gear. With the changes in root widths Lewis form factors of both pinion tooth and gear tooth are altered.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Addendum Modification (Contd....)

Thus by selecting a suitable '+' '-' correction we can reduce the difference in $S_d Y$ for pinion and gear in Lewis formula, as follows, making the pinion and gear teeth of almost equal strength.

$$m = \sqrt[3]{\left(\frac{2T \cos \beta}{Z \psi} \right) \frac{1}{S_d Y}}$$

It optimizes not only the gear sizes but also increases the life of a gear set.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S_o - Plus and S & S_o - Minus Correction

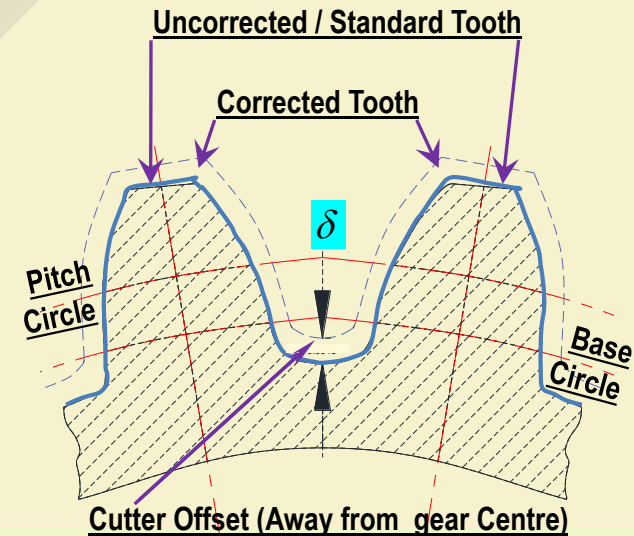
S_o is '+' '-' S correction is without change in centre distance.

S / S_o '+' (Plus) Correction

This means that in S_o correction, if a gear is given '+' correction (to the pinion) then the mating gear is to be given '-' correction by same amount to keep the centre distance unaltered.

'+' correction in (Both External & Internal) Toothed Gear means cutter offset with more CD

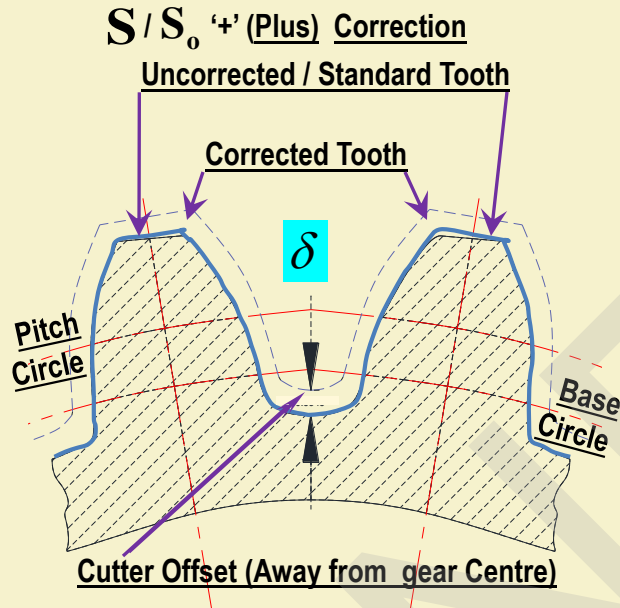
(a) shows '+' correction in a tooth .



Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S_o - Plus and S & S_o - Minus Correction (Contd...)

Correction factor 'x' :



The amount S & S_o '+' (plus) and S & S_o '-' (minus) correction is defined by:

$$\delta = \pm xm$$

Where,

δ is the amount of profile shift or correction factor,

m is the module, and

x is the profile shift or correction factor.

Bibliography

1. **G. M. Maitra, Handbook of Gear Design. (2nd. edn., 1989 ISBN 0-07-460237-3), McGraw-Hill pub. Co. ltd. India.**
2. **Extract from DIN 3960**

Thank You



Module-07

Introduction to Involute Gear Tooth Correction

Lecture-34

Involute Spur Gear Tooth Correction : Part-II

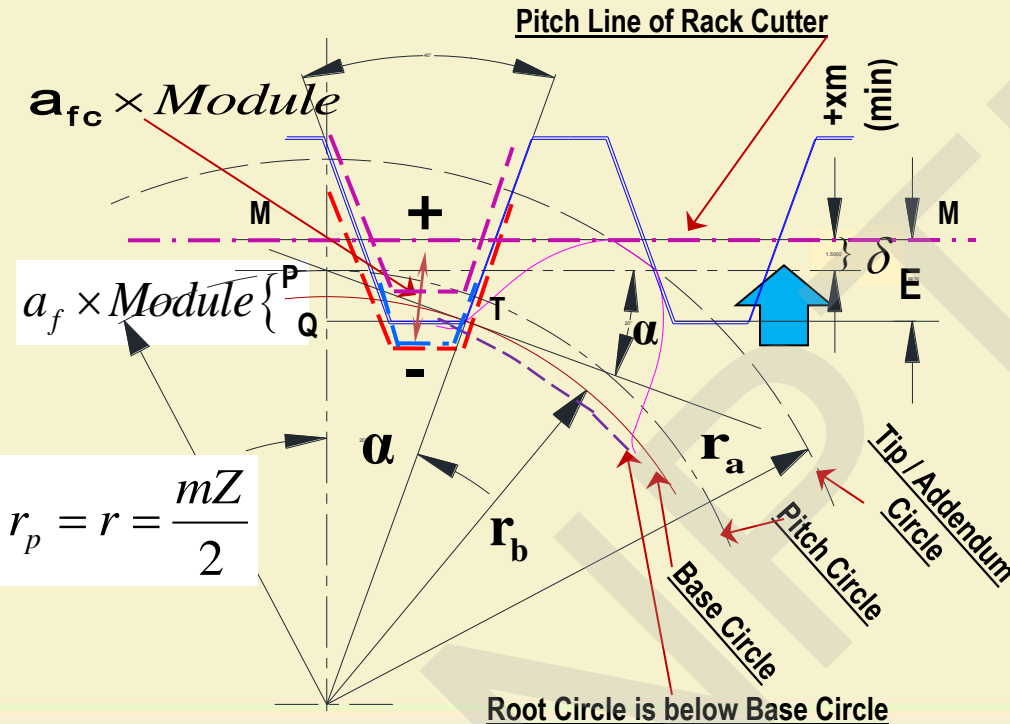


Outline of the Lecture

- **Concept of Involute Gear Tooth Correction**
 - **Amount of Profile Shift (Tooth Correction)**
 - **Recommendation on Amount of Profile Shift ,
Minimum No. of teeth at different operating Condition**
 - **Characteristics of Corrected Gears.**
 - **Effect of amount of Profile shift on Tooth Shape, Under cut etc.**
 - **Forms of Tooth , Under cut etc.**

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S₀- Plus and S & S₀- Minus Correction (Contd...)



Amount of Correction :

$$\delta = \pm xm$$

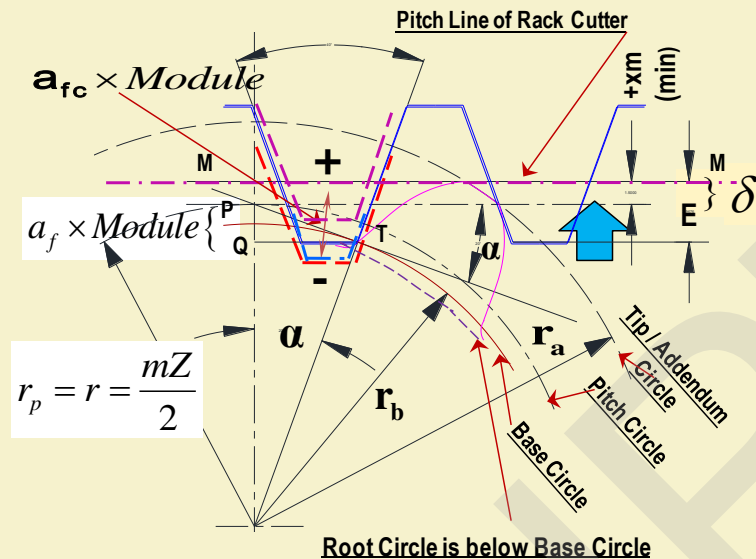
S₀ Means no change in Centre distance.

Or, in other words mating gears have '+' (usually pinion) & '-' (usually Gear) correction.

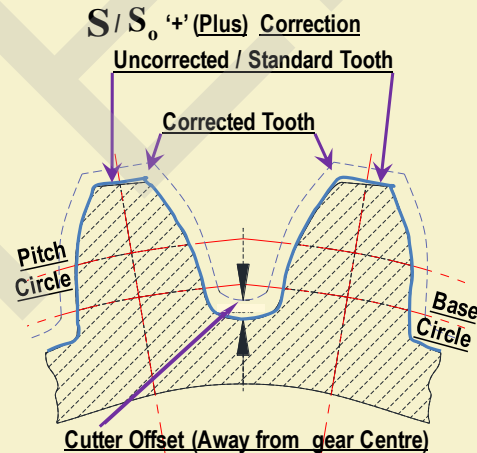
S Correction with change in Centre distance.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S₀- Plus and S & S₀- Minus Correction (Contd...)



This means that mating gears have '+' & '-' corrections not of equal amount.

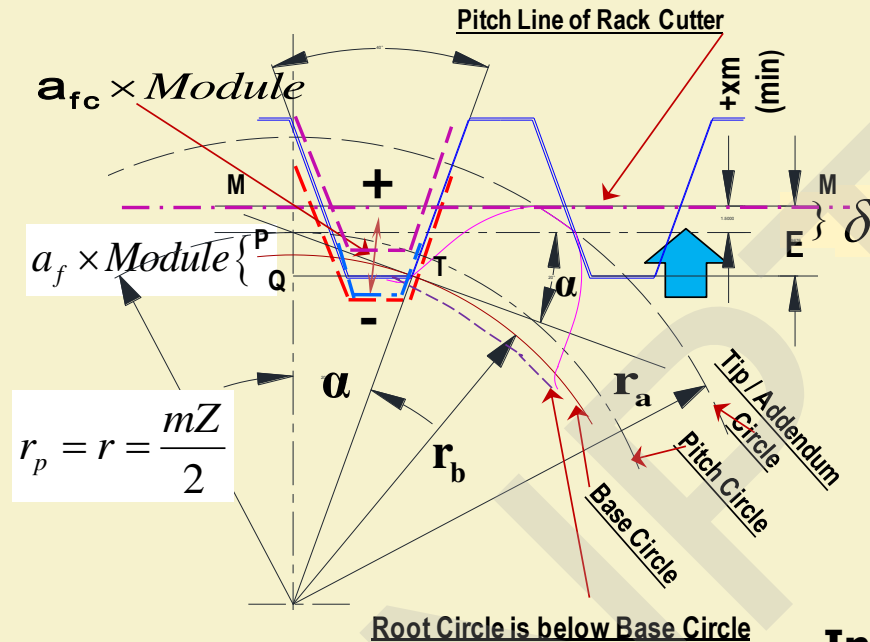


Or, both may have '+' or '-' corrections.

Or, either of the mating gears has '+' or '-' corrections and other has no correction.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S₀- Plus and S & S₀- Minus Correction (Contd...)



'+' (plus) sign is for the '+' correction i.e., outward shift of cutter.

This means that in case of external tooth the cutter is not fed to its full depth.

In case of internal gear it is vice versa.

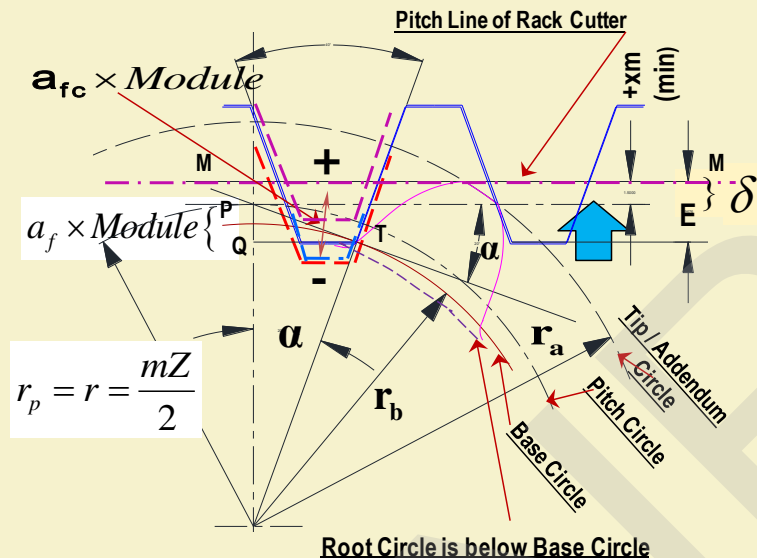
However, in both cases the centre distance between the cutter and the gear being cut (or the reference distance if the cutter is basic rack) is larger by an amount of xm than the standard one.

In case of '-' correction these are reversed.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S_o - Plus and S & S_o - Minus Correction (Contd...)

Minimum Number of Tooth with Correction (Profile Shift) :



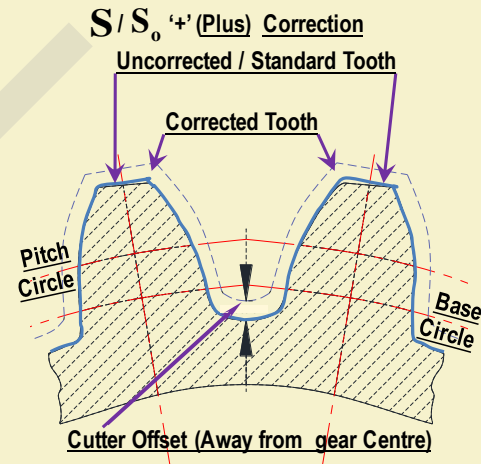
Referring to Illustrations:

$$\sin \alpha = \frac{PQ}{PT} = \frac{m - xm}{PT}$$

But, $PT = r_p \sin \alpha$

Therefore, $m(1 - x) = \frac{mZ}{2} \sin^2 \alpha$

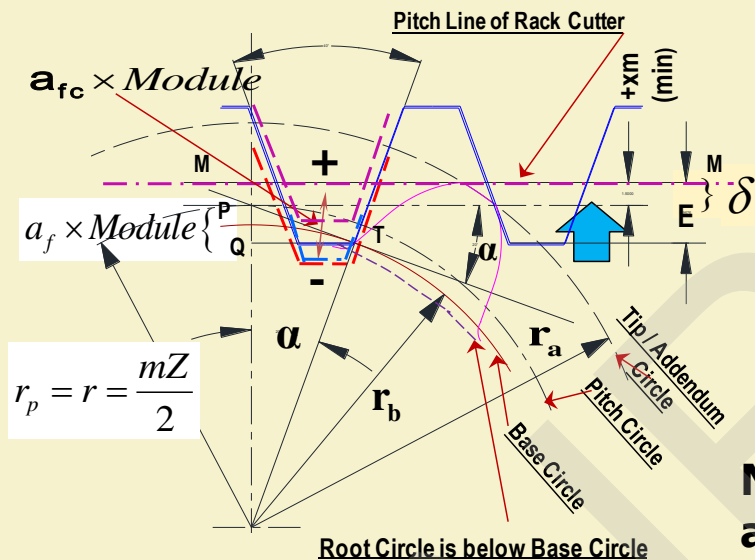
or, $(1 - x) = \frac{Z}{(2/\sin^2 \alpha)} \Rightarrow 1 - x = \frac{Z}{Z_{\min}}$



Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S_0 - Plus and S & S_0 - Minus Correction (Contd...)

Minimum Number of Tooth with Correction (Profile Shift) :

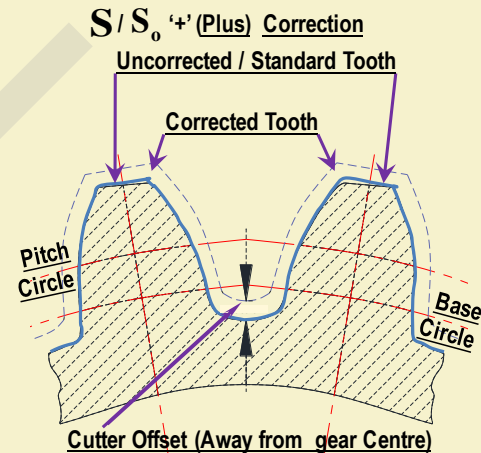


$$Z_{\min} = Z_C$$

Which we have already defined earlier as:

$$Z_c = \frac{2a_f}{\sin^2 \alpha}$$

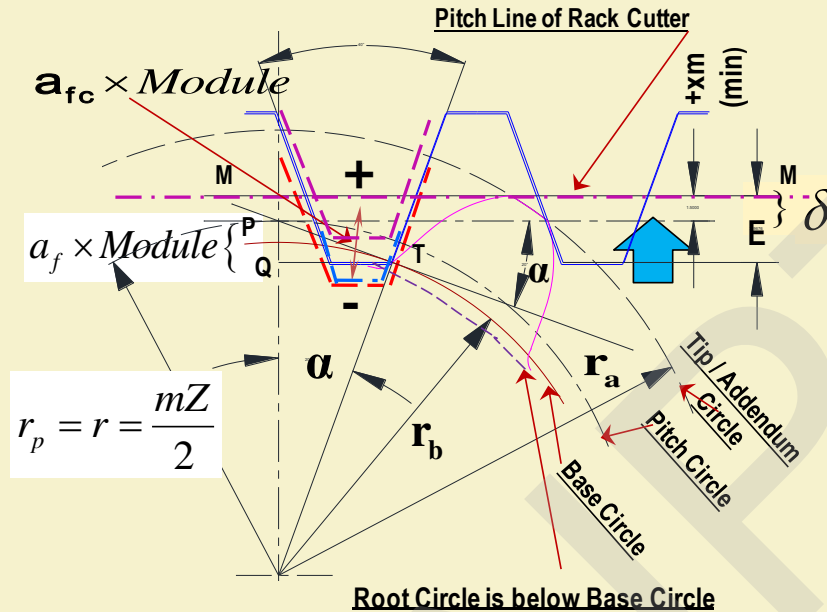
Now for standard gears with addendum factor = 1:



$$\Rightarrow Z_{\min} = \frac{2}{\sin^2 \alpha}$$

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S_0 - Plus and S & S_0 - Minus Correction (Contd...)



Considering, $1 - x = \frac{Z}{Z_{\min}}$

Or, $x = \frac{Z_{\min} - Z}{Z_{\min}}$

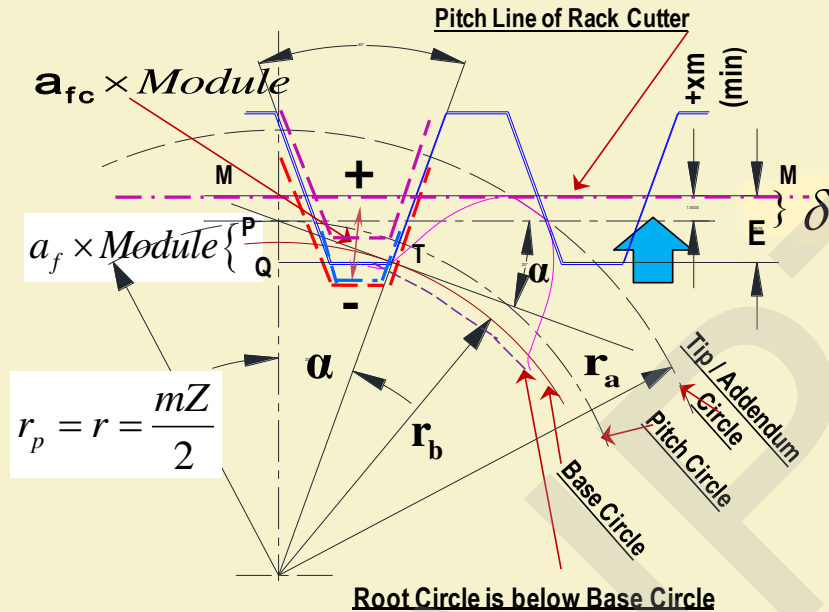
In practice Z can be taken less than that calculated by equation-

$$\frac{2}{\sin^2 \alpha} = Z_{\min}$$

ignoring the minor deviation in tooth profile near the root.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S₀- Plus and S & S₀- Minus Correction (Contd...)



For an example in case of 20° involute $Z_{\min}$ or Z_C is calculated as 17 (17.097).

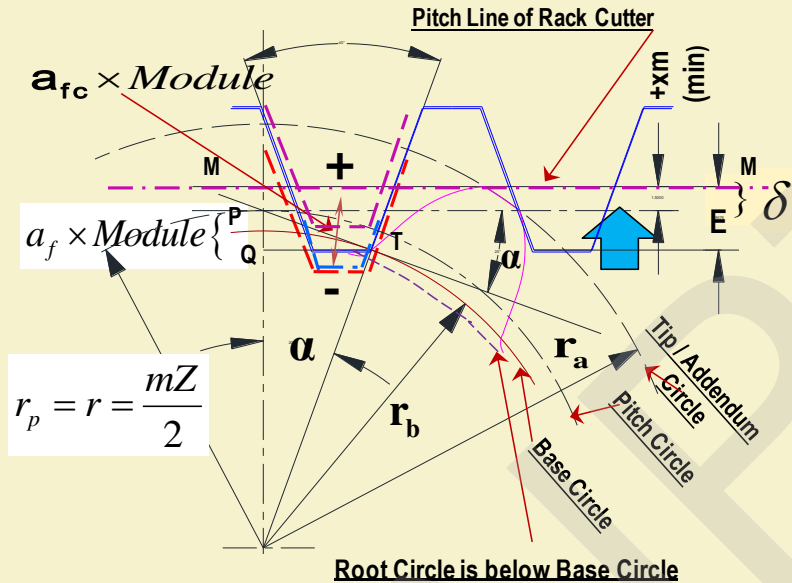
But in practice even without correction it is taken as $Z_{\min p}$, which is as follow:

$Z_{\min p} = 12$ for slow speed application,
 $= 14$ for medium speed application,

However, it is
 $= 18$ for high speed application.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S₀ - Plus and S & S₀ - Minus Correction (Contd...)



Therefore, factor x is expressed and used in practice, as:

$$x = \frac{Z_{\min p} - Z}{Z_{\min}}$$

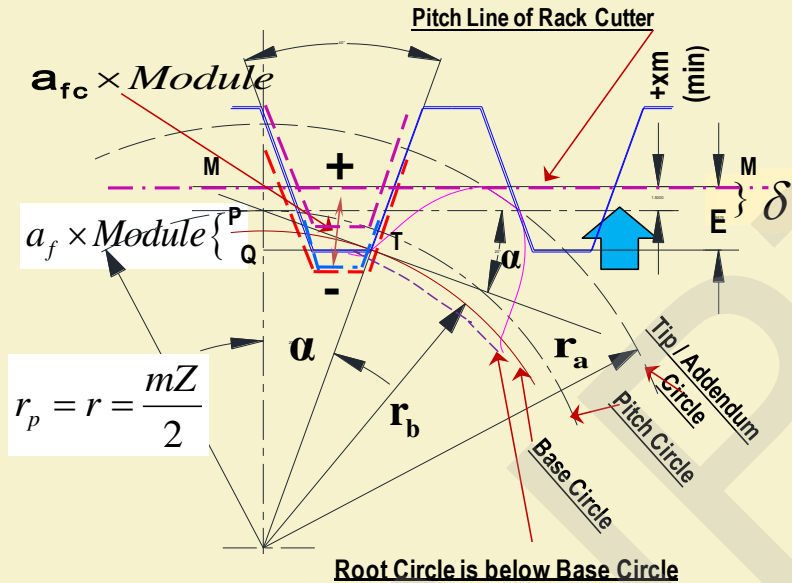
Example: Let Z be 13 (20° pressure angle) and it is of slow speed application.

Then a correction of

$$x = \frac{12 - 13}{17} = -0.0588$$

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

S & S_0 - Plus and S & S_0 - Minus Correction (Contd...)



'-' sign indicates that more depth of cut can be given although the gear tooth is already has undercut.

If the same gear is to be used for high speed application then minimum correction is required for smooth running.

$$x = \frac{18-13}{17} = +0.294$$

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Characteristics of Corrected Gears :

- **Avoidance of undercutting,**
- **Attainment of a predetermined modified centre distance,**
- **To increase root and flank strength of tooth,**
- **Betterment of sliding and contact relations, and**
- **To shift the beginning of effective profile away from the base circle.**

With the gear correction (both types) the base circle does not change.

Therefore, all calculations can still be referred to the base circle.

Also, the circular pitch (at standard pitch circle) remains same, although the thickness of the corrected tooth at standard pitch circle is different from its standard value which is $\pi m / 2$.

Effect of Amount of Correction on Involute Spur Gear Tooth :

Characteristics of Corrected Gears (Contd...):

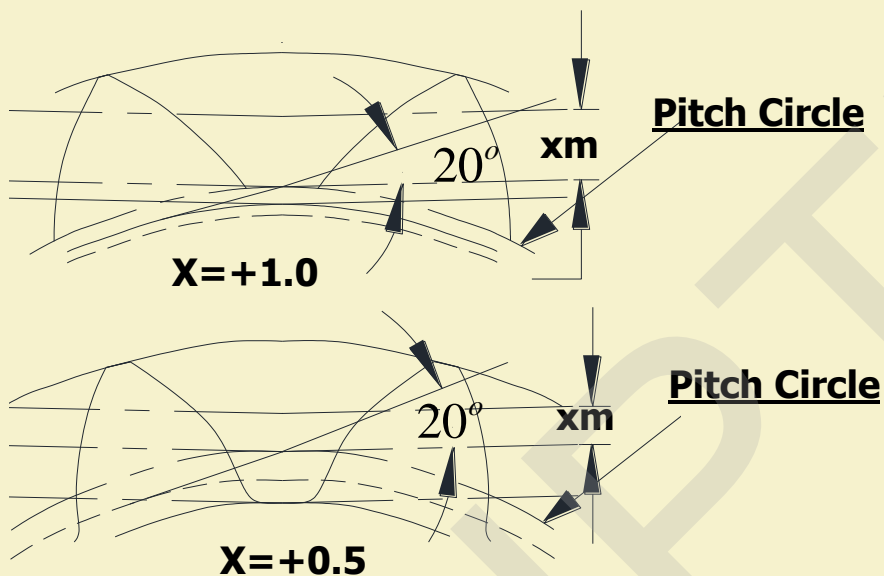
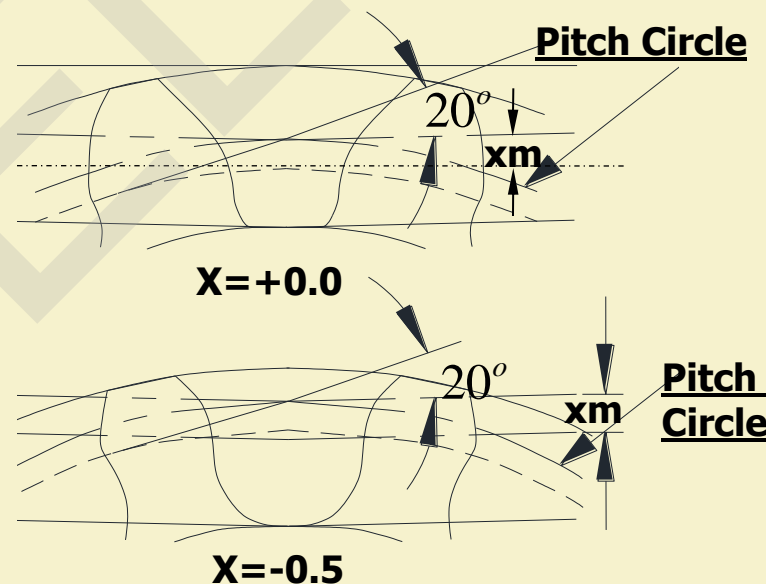
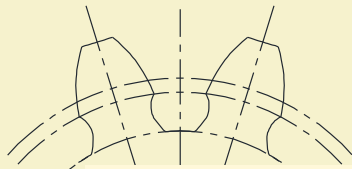


Illustration show the effects of amount of correction on tooth shape ($Z=17$).

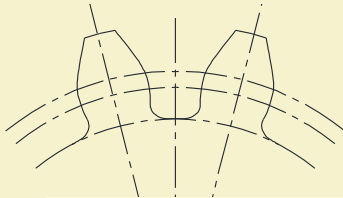
Forms of Corrected and Undercut Involute Spur Gear Tooth :



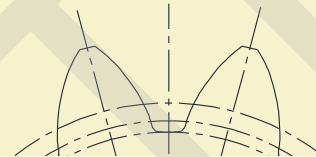
(a) $Z=10, x=0$



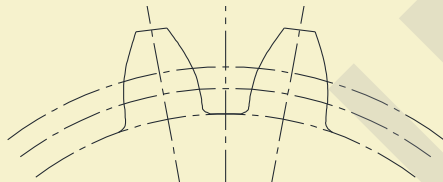
(b) $Z=10, x=+0.5$



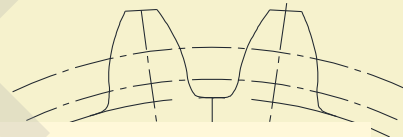
(c) $Z=14, x=0$



(d) $Z=14, x=+0.5$



(e) $Z=17, X=0$



(f) $Z=24, x=0$

Illustrations shows the changes in tooth form 20° Involute with the plus-minus correction.

Bibliography

1. **G. M. Maitra, Handbook of Gear Design. (2nd. edn., 1989 ISBN 0-07-460237-3), McGraw-Hill pub co ltd India.**
2. **Extract from DIN 3960**



Thank You



Module-07

Introduction to Involute Gear Tooth Correction

Lecture-35

Involute Spur Gear Tooth Correction : Part-III



Outline of the Lecture

- **Characteristics of Involute Toothed Corrected Gears (Contd...)**
 - **Interference and Undercut**
 - **Ideal and Practical Critical Number of Teeth and Peaking of Tooth**
 - **Peaking Limit**
 - **Modified parametric values with 'S' correction & Centre distance modification**
 - **Working Pressure Angle**
 - **Arc Tooth Thickness at Standard Pitch Circle**
 - **Arc Tooth Thickness and Space Thickness at Working Pitch Circle**

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Characteristics of Corrected Gears (Contd...) :

Interference and Undercut

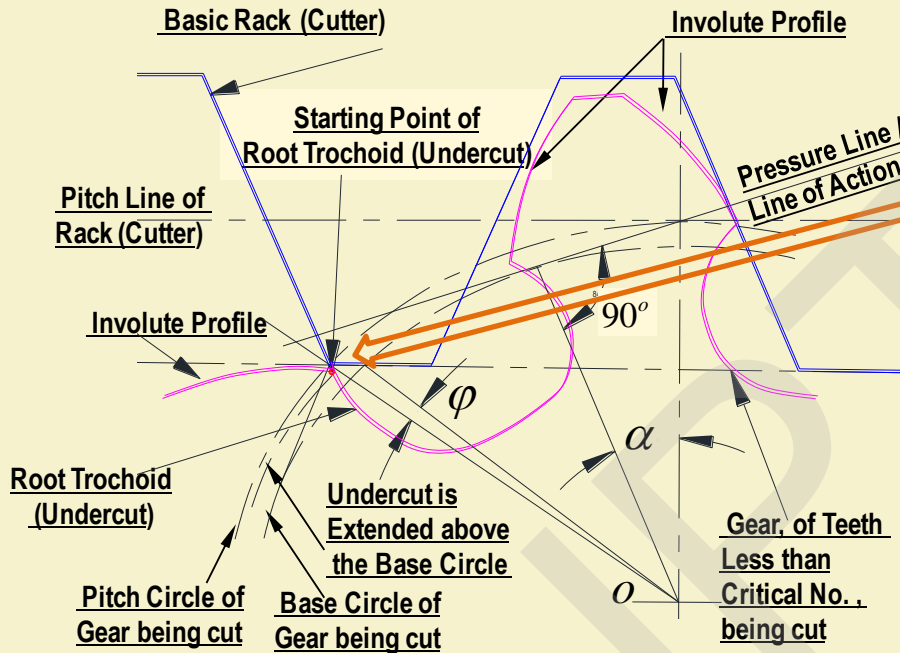


Fig. 1 : Interference and Undercut

A Gear of teeth number less than the critical minimum number ($2a_f / \sin^2 \alpha$) is being cut.

The undercut is extended above the base circle i.e., it also cuts the beginning portion of involute profile.

A portion of involute tooth tip of mating gear loses its conjugacy resulting in gear noise and intolerable dynamics.

Plus '+' correction improves, if not eliminates such problem of tooth root.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Characteristics of Corrected Gears (Contd...) : Z_c , Z_{min} and Peaking Limit :

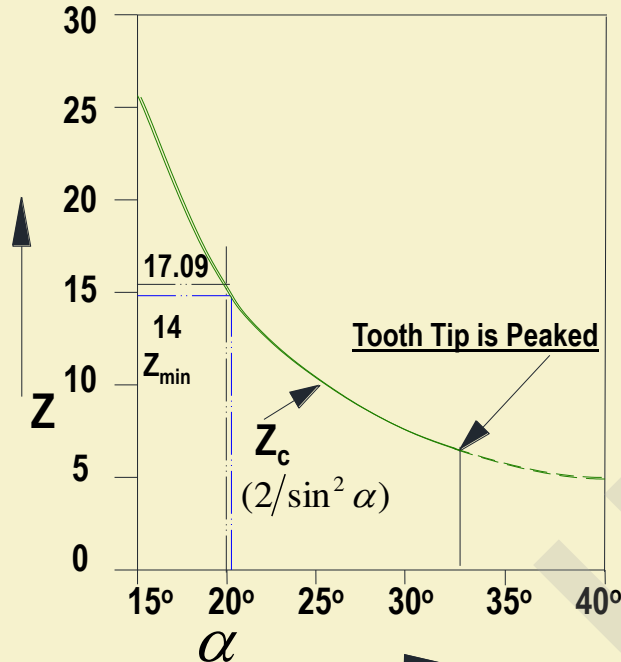


Fig. 2 : Critical (Minimum) Number of Teeth At Different Pressure Angle

Critical minimum number of teeth to avoid undercut ($Z_c = 2/\sin^2 \alpha$) is shown in the illustration.

Tooth Tip become zero width at its Z_c when pressure angle is about 33° .

With + correction peaking reached in external involute tooth after a limit.

One effect of the positive S correction is to make the tooth more and more pointed as the correction factor increases.

It is called 'peaking' of Tooth.

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Characteristics of Corrected Gears (Contd...) : Peaking limit:

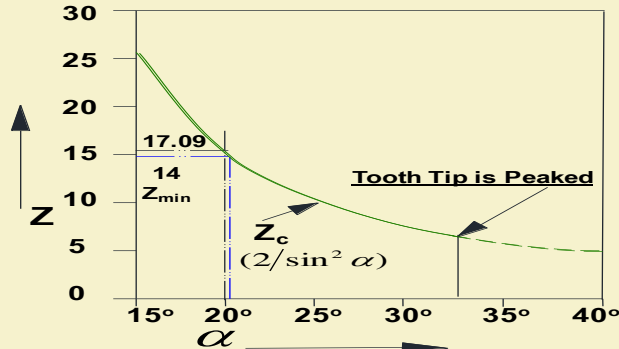


Fig. 2 : Critical (Minimum) Number of Teeth At Different Pressure Angle

Usually a top land width below “0.4 times module” weakens the tooth tip.

Fig. 3 shows the peaking limits with the decrease in teeth number, with respect to pressure angle, for standard gears.

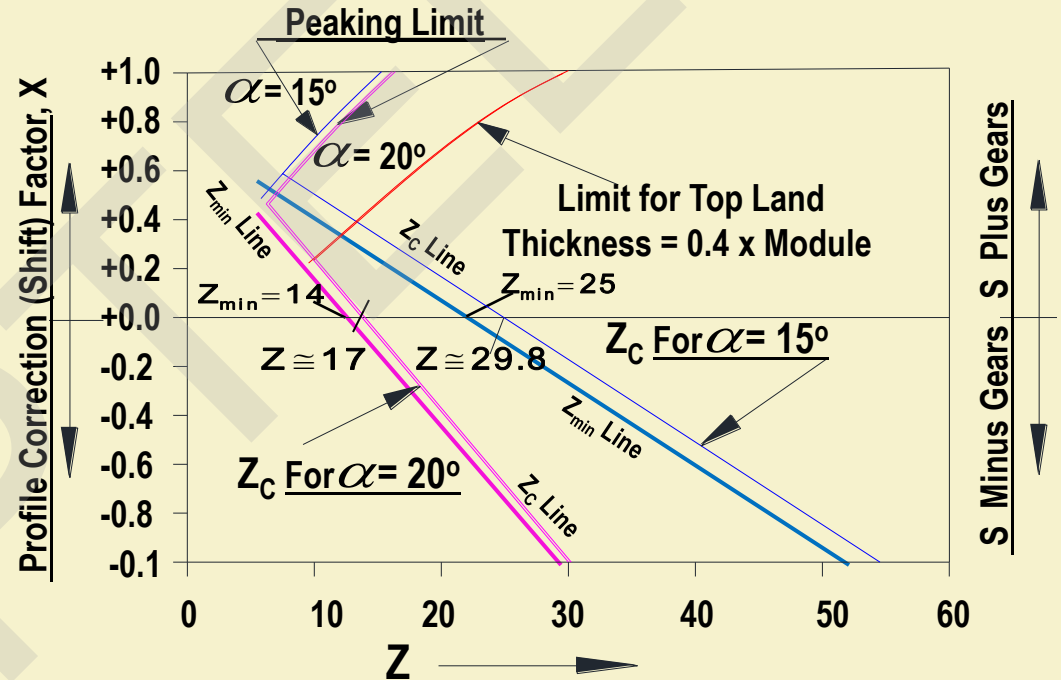


Fig. 3 : Amount of Profile Shift, Critical No. of Teeth & Peaking Limit

Introduction to Involute Spur Gear Tooth Correction (Contd...) :

Characteristics of Corrected Gears (Contd...) : Peaking limit (Contd.....):

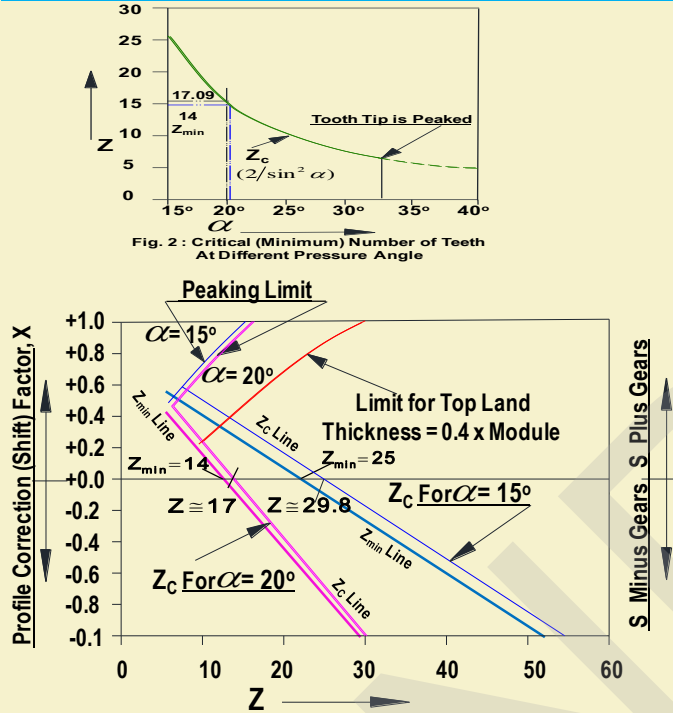


Fig. 3 : Amount of Profile Shift, Critical No. of Teeth & Peaking Limit

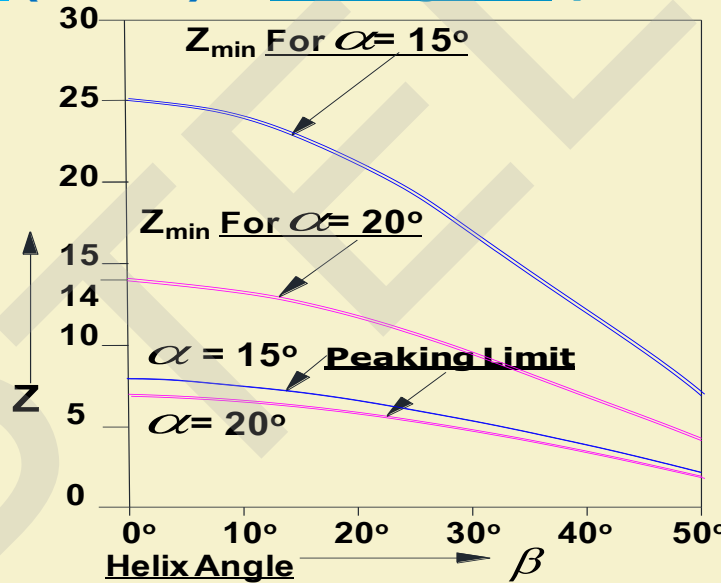


Figure- 4 shows the peaking limits with respect to pressure angle & helix angle.

The possible amount of correction is chosen from all these graphs.

Involute Spur Gear Tooth Correction :

Modified parametric values with 'S' correction & centre distance modification:

Working Pressure Angle α_w :

Let x_1 and x_2 be the 'correction factor' for gear-1 and gear-2 respectively.

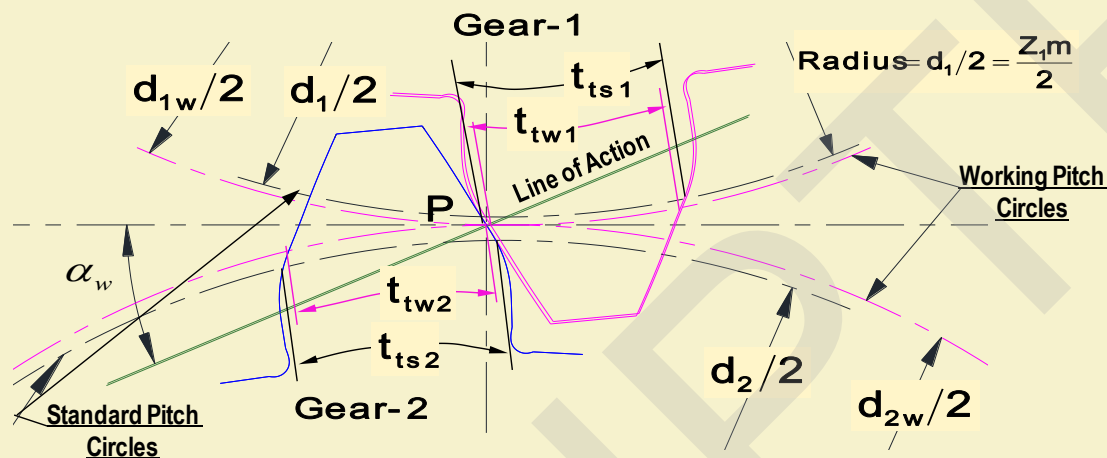


Fig.-5: Geometry for Working Pressure Angle (α_w)

The values of the correction factor may be positive, negative or zero depending on sense and amount of correction.

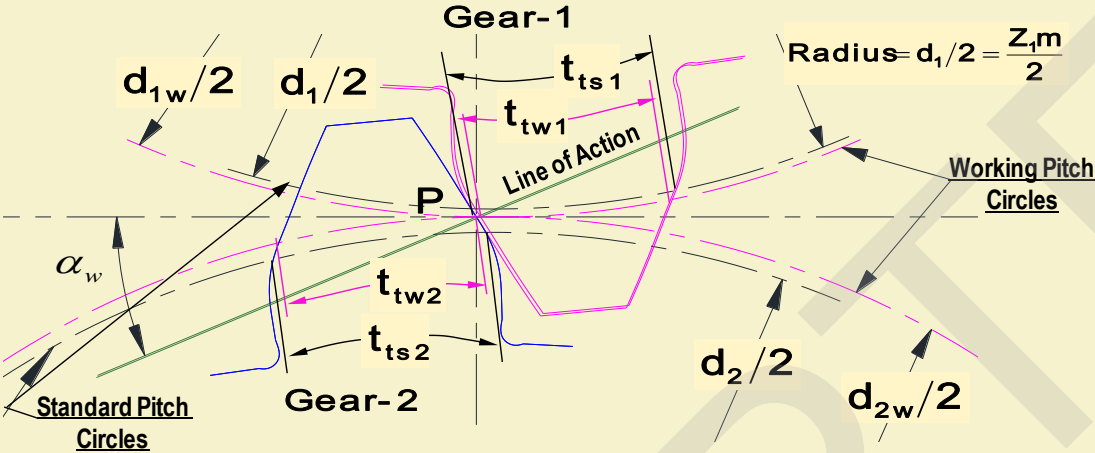
The modified centre distance A is calculated as:

$$A = A_o + (x_1 + x_2)m$$

Involute Spur Gear Tooth Correction :

'S' correction & centre distance modification:

Working Pressure Angle α_w (Contd....) :



x_1 and x_2 be the 'correction factor' for gear-1 and gear-2 respectively.

Where, the centre distance A_o with standard uncorrected gear is expressed as:

$$A_o = (Z_1 + Z_2)m / 2$$

As with the corrections, base circles do not change.

Fig.-5: Geometry for Working Pressure Angle (α_w)

Therefore,

$$A_o = (d_1 + d_2) / 2 = \frac{1}{\cos \alpha} (r_{b1} + r_{b2})$$

$$\text{Or, } (r_{b1} + r_{b2}) = A_o \cos \alpha$$

Involute Spur Gear Tooth Correction :

'S' correction & centre distance modification:

Working Pressure Angle α_w (Contd....) :

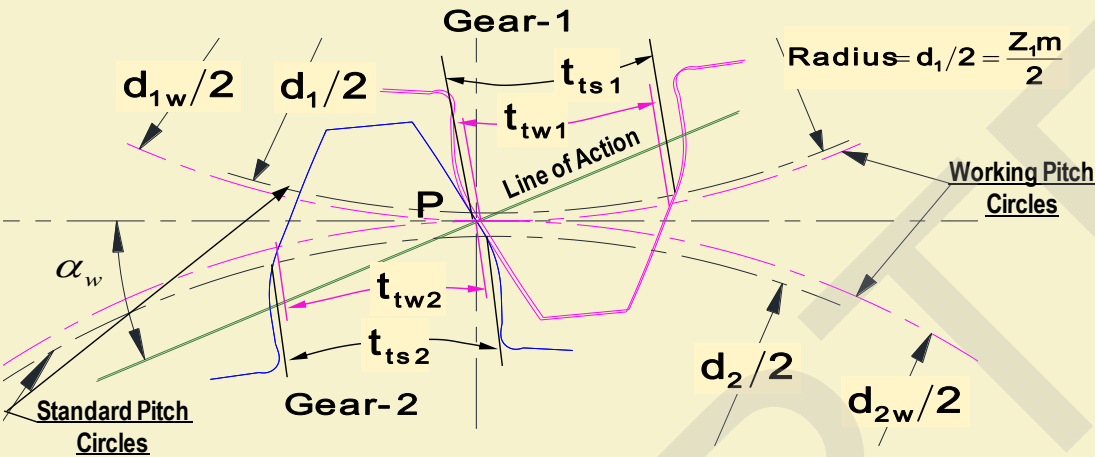


Fig.-5: Geometry for Working Pressure Angle (α_w)

Similarly,

$$A = (d_{1w} + d_{2w}) / 2$$

$$= \frac{1}{\cos \alpha_w} (r_{b1} + r_{b2})$$

$$\text{or, } (r_{b1} + r_{b2}) = A \cos \alpha_w$$

$$\text{Therefore, } A \cos \alpha_w = A_0 \cos \alpha$$

And finally the working pressure angle α_w , is expressed as : $\alpha_w = \cos^{-1} \left(\frac{A_0 \cos \alpha}{A} \right)$

Involute Spur Gear Tooth Correction : 'S' correction & centre distance modification:

Arc Tooth Thickness at Standard Pitch Circle, :

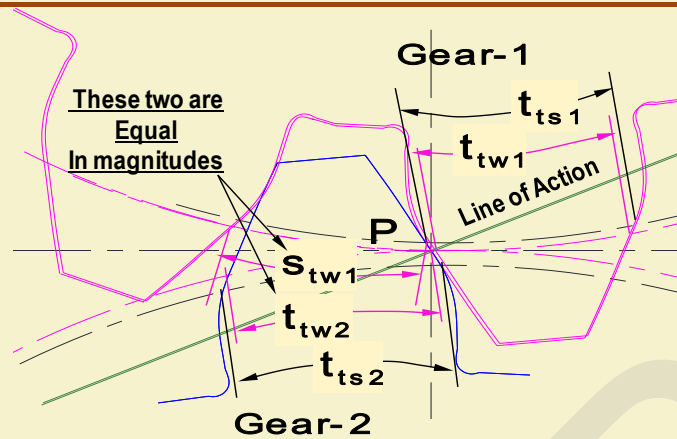


Fig.- 6 : Tooth and Space Thicknesses

On working pitch circles

the arc length of gap s_{tw1} on working pitch circle (working or standard) between two consecutive teeth of a gear is equal to the arc tooth thickness t_{tw2} on its own pitch circle of the mating gear, in case of backlash free mesh.

Or in other words, $t_{tw2} = s_{tw1}$ and $t_{tw1} = s_{tw2}$

$$\text{For Gear- 1, } t_{ts1} = \frac{\pi m}{2} + 2x_1 m \tan \alpha$$

$$\text{For Gear- 2, } t_{ts2} = \frac{\pi m}{2} + 2x_2 m \tan \alpha$$

Working Circular Pitch, p_{cw} :

It is to be noted that as per gearing law,

Involute Spur Gear Tooth Correction : 'S' correction & centre distance modification:

Circular Pitch and Tooth Thicknesses at Working pitch Circle:

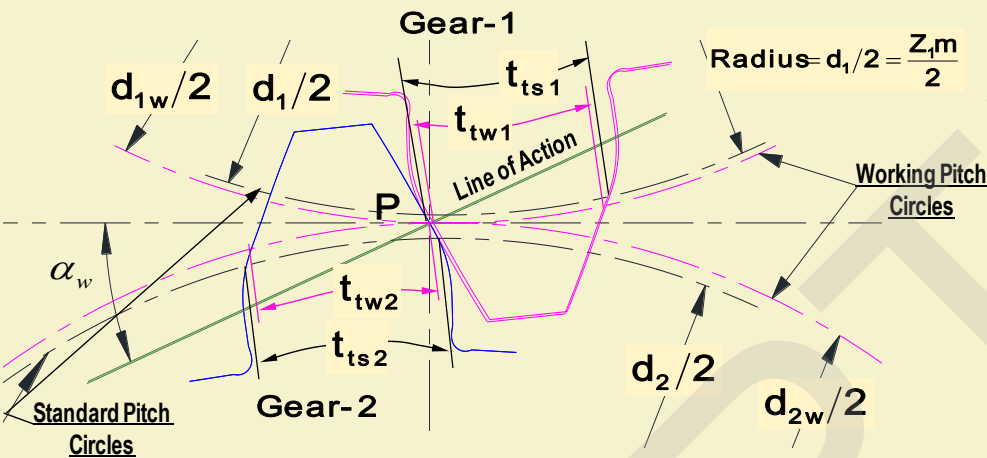


Fig.-5 : Geometry for Working Pressure Angle (α_w)

The circular pitch at working pitch circle,

$$p_{cw} = t_{tw1} + s_{tw1} = t_{tw2} + s_{tw2}$$

$$= \frac{2\pi r_{w1}}{Z_1} = \frac{2\pi r_{w2}}{Z_2}$$

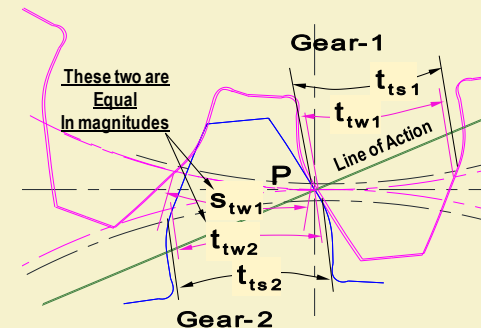


Fig.-6 : Tooth and Space Thicknesses
On working pitch circles

Involute Spur Gear Tooth Correction : 'S' correction & centre distance modification:

Working Circular Pitch, p_{cw} (contd....) :

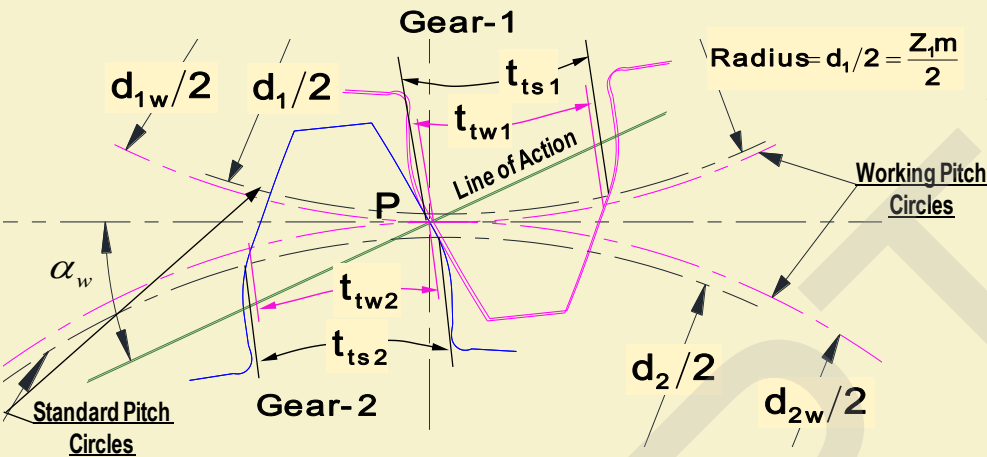


Fig.-5 : Geometry for Working Pressure Angle (α_w)

It is to be noted that as per gearing law, the gear ratio i is expressed as:

Therefore, as $S_{tw1} = t_{tw2}$

$$t_{tw1} + t_{tw2} = \frac{2\pi r_{w1}}{Z_1} = \frac{2\pi r_{w2}}{Z_2}$$

$$i = \frac{Z_2}{Z_1} = \frac{r_{w2}}{r_{w1}}$$

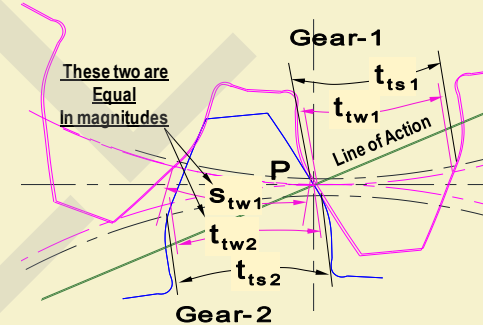


Fig.-6 : Tooth and Space Thicknesses On working pitch circles

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Module-07

Introduction to Involute Gear Tooth Correction

Lecture-36

Involute Spur Gear Tooth Correction : Tutorial (Workout Example)



Outline of the Lecture

- **Workout Example of Gear Tooth Correction (Profile Shift)**
 - **Problem- Leading to Non Standard Centre Distance**
 - **Selection of Teeth Numbers, Working Centre Distance etc.**
 - **Calculation for Working Pressure Angle**
 - **Some Useful Formulae**
 - **Estimation of Profile Shifts for Gear-1 and Gear 2**
 - **Calculation for Topping.**

Involute Spur Gear Tooth Correction :

Modified parametric values with 'S' correction & centre distance modification:

Numerical Example – 1 :

A straight tooth spur gear set of $(\alpha =) 20^0$ involute teeth has a gear ratio of $i = 3:2 = 1.5$, centre distance $A = 210 \text{ mm}$ and module $m = 10 \text{ mm}$.
Select the teeth number and necessary correction keeping i and A exact.

Solution :

$$A = (Z_1 + Z_2)m / 2$$

$$\text{Now, } i = \frac{Z_2}{Z_1} = \frac{3}{2}$$

$$\text{or, } (Z_1 + Z_2) = 210 \times 2 / 10 = 42$$

Solution of teeth numbers, Z_1 & Z_2 expressed by above two equations does not lead to any whole numbers.

Involute Spur Gear Tooth Correction - Workout Example (Contd..):

Solution (Contd...):

We consider the S correction and select the teeth numbers as:

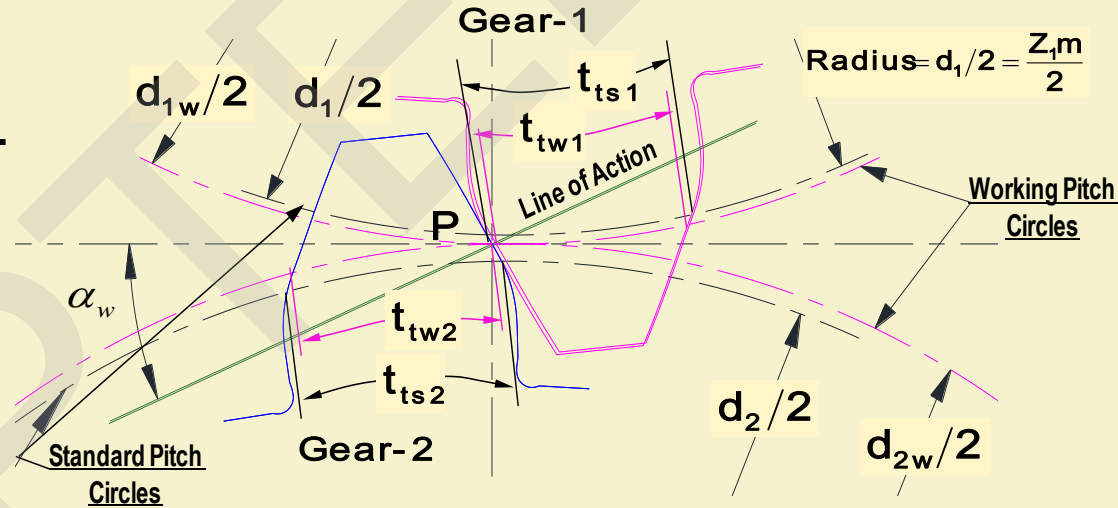
$$Z_1 = 16 \quad \text{and} \quad Z_2 = 24$$

$$\text{for which} \quad (Z_1 + Z_2) = 40.$$

This is the nearest possible combination with minimum correction and smaller number of teeth in pinion.

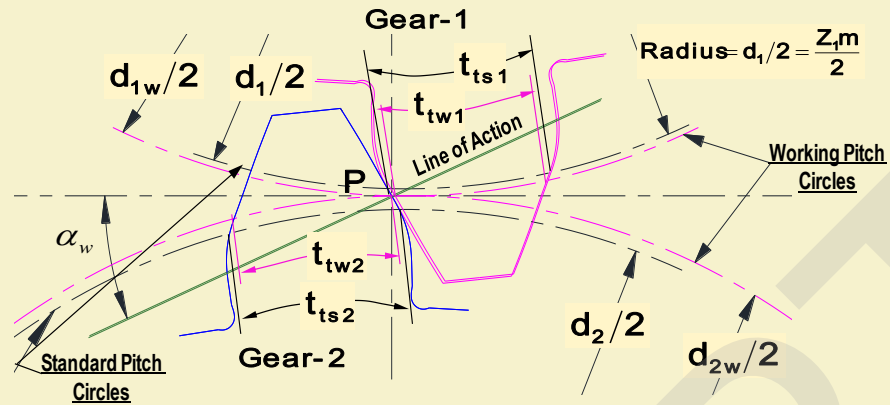
Without correction the centre distance A_0 is calculated as:

$$A_0 = 40 \times 10 / 2 = 200 \text{ mm}$$



Geometry for Working Pressure Angle (α_w)

Involute Spur Gear Tooth Correction – Workout Example (Contd..) :



Geometry for Working Pressure Angle (α_w)

With S type correction the centre distance is to be made 210 mm.

For which the working pressure angle is estimated as follows.

As with the corrections base circles do not change. Therefore, using eqn.

$$\alpha_w = \cos^{-1} \left(\frac{A_0 \cos \alpha}{A} \right)$$

and substituting data we get working pressure angle α_w , on CD modification:

$$\alpha_w = \cos^{-1} \left(\frac{A_0 \cos \alpha}{A} \right) = \cos^{-1} \left(\frac{200 \cos 20^\circ}{210} \right) = 26^\circ 30'$$

Involute Spur Gear Tooth Correction :

Modified parametric values with 'S' correction & centre distance modification:

Some Useful Formulae in summery form :

Table-1	
Standard centre distance	$A_0 = m \frac{Z_1 + Z_2}{2}$
Actual centre distance (after pushing)	$A = A_0 \frac{\cos \alpha}{\cos \alpha_w} = m \frac{Z_1 + Z_2}{2} \frac{\cos \alpha}{\cos \alpha_w}$
Working pressure angle	$\text{inv } \alpha_w = 2 \frac{X_1 + X_2}{Z_1 + Z_2} \tan \alpha + \text{inv } \alpha$
Sum of profile correction factors	$x_1 + x_2 = (Z_1 + Z_2) \frac{\text{inv } \alpha_w - \text{inv } \alpha}{2 \tan \alpha}$

Involute Spur Gear Tooth Correction :

Modified parametric values with 'S' correction & centre distance modification:

Some Useful Formulae in summery form :

Table-2 : Dimensions for S- gearing		
Description	Pinion	Gear
Number of teeth	Z_1	Z_2
Pitch circle diameter	$d_1 = Z_1 m$	$d_2 = Z_2 m$
Tip circle diameter (with topping)	$d_{a1} = 2(a + m - x_2 m) - d_2$	$d_{a2} = 2(a + m - x_1 m) - d_1$
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1) m$	$d_{f2} = d_2 - 2(1.25 - x_2) m$
Tooth thickness on pitch circle	$s_1 = \frac{p}{2} + 2 x_1 m \tan \alpha$	$s_2 = \frac{p}{2} + 2 x_2 m \tan \alpha$
Topping	$y m = a_0 + (x_1 + x_2) m - a$	

Involute Spur Gear Tooth Correction – Workout Example (Contd..) :

Using involutometry:

$$inv\alpha_w = \frac{2}{(Z_1 + Z_2)} \tan \alpha (x_1 + x_2) + inv\alpha$$

Substituting the values :

$$inv\alpha_w = \tan \alpha_w - \alpha_w = \tan(26^\circ 30') - \frac{26^\circ 30'}{180^\circ} \pi = 0.4985816081 - 0.4625122518$$

$$\therefore inv\alpha_w = 0.03606935627$$

Similarly, $inv\alpha = 0.01490438387$

[Note: In calculating involute of any angle at least five decimal places of all components in equations are to be considered.]

Involute Spur Gear Tooth Correction – Workout Example (Contd..) :

Substituting the values in $inv\alpha_w = \frac{2}{(Z_1 + Z_2)} \tan \alpha (x_1 + x_2) + inv\alpha$

we get: $(x_1 + x_2) = 1.163$

For such a higher value and as the centre distance is being increased in tooth correction both the gear and pinion must be given S '+' correction.

Also, considering equal root stress we use eqn. for distribution of corrections and we put x_1 in pinion and x_2 in gear respectively.

Involute Spur Gear Tooth Correction – Workout Example (Contd..) :

$$x_1 = \frac{x_1 + x_2}{i + 1} + 0.5 \frac{i - 1}{i + 1} = \frac{1.163}{1.5 + 1} + 0.5 \frac{1.5 - 1}{1.5 + 1} = 0.5652; \quad \& \quad x_2 = 1.163 - 0.5652 = 0.5978$$

Topping is calculated as:

$$y_m = A_o + (x_1 + x_2)m - A = 1.163 \times 10 - 10 = 1.63 \text{ mm}$$

Therefore, the addendum i.e., blank diameter of the pinion:

$$d_{a1} = d_1 + 2m + 2x_1m - 2y_m = 188.044 \text{ mm}$$

Similarly, the addendum i.e., blank diameter of the gear

$$d_{a2} = d_2 + 2m + 2x_2m - 2y_m = 268.696 \text{ mm}$$

Thank You



Module-07

Introduction to Involute Gear Tooth Correction

Lecture-37

Involute Spur Gear Tooth Correction : Tutorial (Workout Example-2)



Outline of the Lecture

- **A Problem on Internal (Involute) Gear Tooth Correction**
 - **Calculation of Backlash**
 - **Working Pressure Angle**
 - **Corrected Pinion Tooth Tip Thickness**
 - **Internal (Ring) Gear Tooth Tip Thickness**
- **Contact Ratio**

Problem on Gear Tooth Correction in Internal Gearing :

Numerical Example – 2 :

In an internal gear drive with 20° involute straight tooth spur gear set numbers of teeth are 50 and 40 and module is 5 mm. Addendum factor (with respect to uncorrected gear) is 0.8 for both.

An amount of $-0.06 S_o$ correction is given to the pinion only.

Find out: (i) backlash,

**(ii) tooth tip thicknesses of both gear and pinion,
and (iii) contact ratio.**

Problem on Gear Tooth Correction in Internal Gearing :

Solution :

Given Data :

Ring (Internal toothed) Gear & Pinion (External Toothed) :-

$$Z_g = 50, \quad Z_p = 40, \quad \alpha = 20^\circ, \quad m = 5 \text{ mm.}$$

$$\text{Addendum factors - } a_{fg} = a_{fp} = 0.8,$$

and Correction factor for Pinion S_o ' - '0.06.

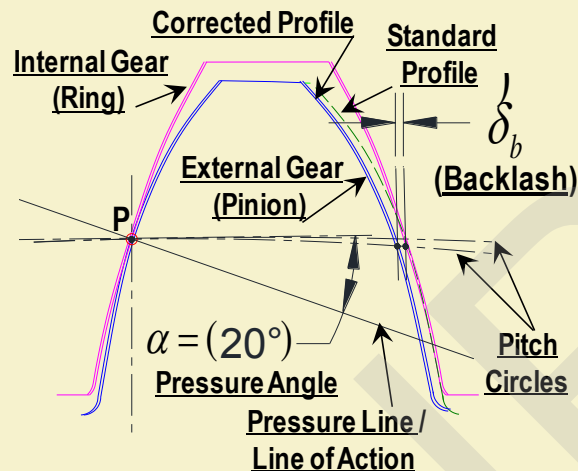
To Find :

Backlash, Tooth tip thicknesses of both Ring Gear & Pinion and Contact ratio.

Problem on Gear Tooth Correction in Internal Gearing : Solution (contd...) :

Backlash :

The Pinion is given negative correction and there is no correction on ring gear. This means that the Pinion tooth is made thinner and there is no change in the centre distance.



An Uncorrected Ring Gear and Corrected Pinion in Mesh with unchanged CD (S_o)

Therefore, the reduction in arc tooth thickness of pinion tooth at pitch circle equals to backlash:

It is calculated as:

$$\begin{aligned}\delta_b &= 2|x_p|m \tan \alpha \\ &= 2 \times 0.06 \times 5 \times \tan 20^\circ \\ &= 0.6 \times 0.36397 \\ &= 0.2184 \text{ mm}\end{aligned}$$

Problem on Gear Tooth Correction in Internal Gearing :

Solution (contd...) :

Pinion tooth tip thickness :

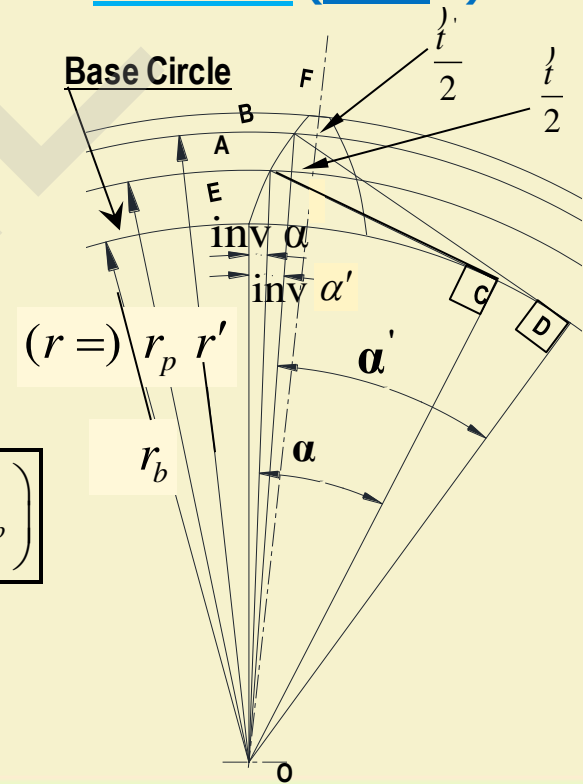
Tooth thickness t' of external gear, at a radius r' is expressed as:

$$t' = 2r' \left(\frac{t}{2r} + \text{inv } \alpha - \text{inv } \alpha' \right)$$

Therefore, tooth tip thickness t_{tip} is expressed as:

$$t_{tip} = 2r_{tip} \left(\frac{t}{2r} + \text{inv } \alpha - \text{inv } \alpha_{tip} \right)$$

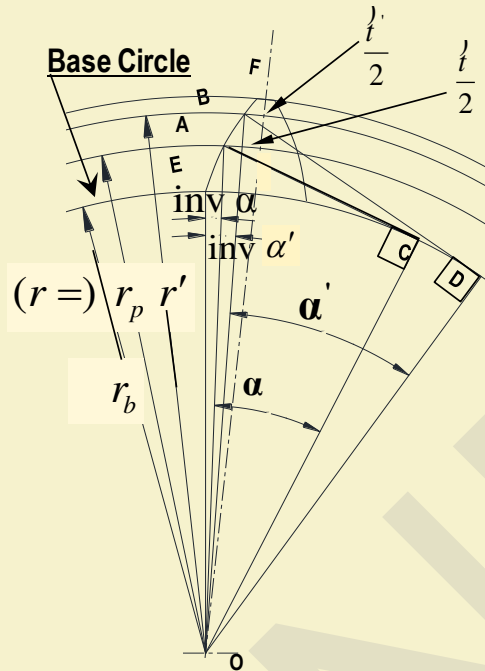
$$r_{tip} = \frac{(Z_p + 2a_{fp})m}{2} = \frac{(40 + 2 \times 0.8) \times 5}{2} = 104 \text{ mm}$$



Problem on Gear Tooth Correction in Internal Gearing :

Solution (contd...) :

Pinion tooth tip thickness (contd....) :



$$r = \frac{(Z_p)m}{2} = \frac{40 \times 5}{2} = 100 \text{ mm}$$

$$t = \frac{\pi m}{2} - \delta_b = \frac{\pi \times 5}{2} - 0.2184$$
$$= 7.854 - 0.2184 = 7.6356 \text{ mm}$$

$$\text{inv } \alpha = \tan \alpha - \alpha = \tan 20^\circ - \frac{20}{180} \pi$$
$$= 0.363970 - 0.349066 = 0.0149$$

Solution (contd...) :

Pinion tooth tip thickness (contd....) :

$$\alpha_{tip} = \cos^{-1} \frac{r}{r_{tip}} \cos \alpha = \cos^{-1} \frac{100}{104} \cos 20^\circ$$

$$= \cos^{-1} \frac{100}{104} \times 0.94 = \cos^{-1} 0.90355$$

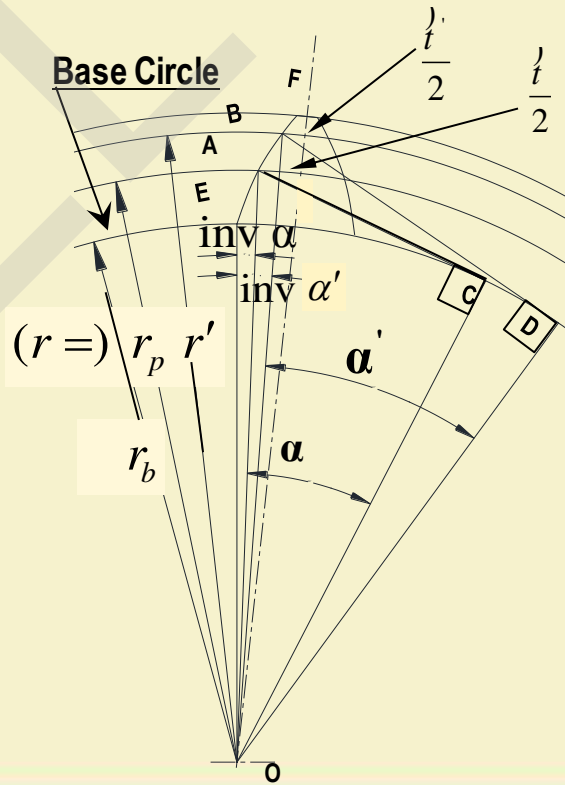
$$= 25.3712^\circ$$

$$inv\ \alpha_{tip} = \tan \alpha_{tip} - \alpha_{tip}$$

$$= \tan 25.3712^\circ - \frac{25.3712}{180} \pi$$

$$= 0.4742196 - 0.442811 = 0.0314082$$

$$t_{tip} = 2 \times 104 \times \left(\frac{7.6356}{2 \times 100} + 0.0149 - 0.0314082 \right) = 4.5073 \text{ mm}$$



Problem on Gear Tooth Correction in Internal Gearing :

Solution (contd...) :

Ring Gear tooth tip thickness :

Tooth thickness t' of external gear, at a radius r' is expressed as:

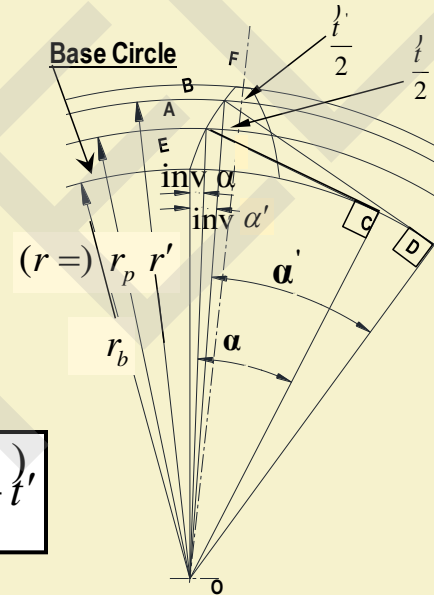
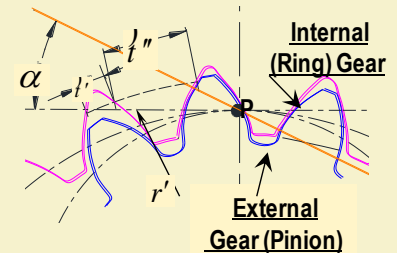
$$t' = 2r' \left(\frac{t}{2r} + \text{inv } \alpha - \text{inv } \alpha' \right)$$

Internal (Ring) Gear

Tooth thickness t'' of internal gear, at a radius r' is expressed as:

$$t'' = \frac{2\pi r'}{Z} - t'$$

Where t' is the thickness of the gap at radius r' of the internal gear.



t' is calculated using the same formula for external gear.

Problem on Gear Tooth Correction in Internal Gearing : [Solution \(contd...\) :](#)

Ring Gear tooth tip thickness (Contd.) :

$$r' = \frac{(Z_g - 2a_{fg})m}{2} = \frac{(50 - 2 \times 0.8) \times 5}{2} = 121 \text{ mm}$$

As the ring gear tooth is not corrected, therefore gap thickness at pitch circle:

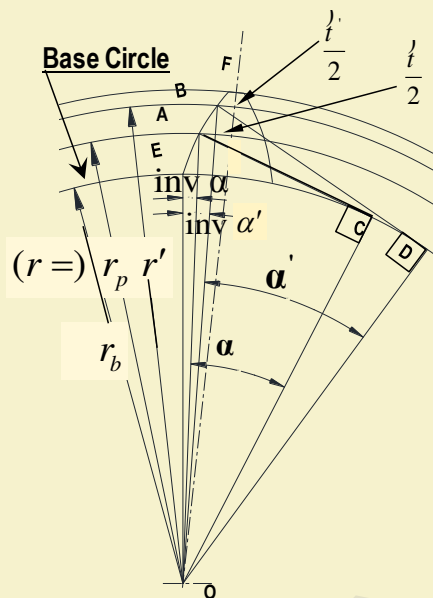
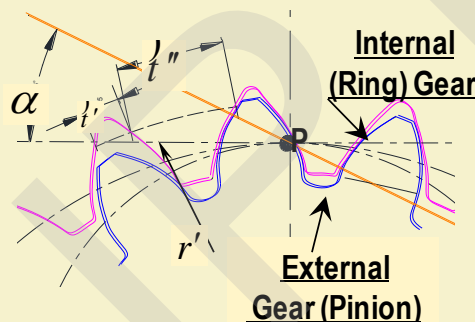
$$t' = \pi m / 2 = \pi 5 / 2 = 7.854 \text{ mm}$$

$$\alpha_{r'} = \alpha_{r_g} = \cos^{-1} \frac{r_{pg}}{r_{tg}} \cos \alpha$$

$$= \cos^{-1} \frac{Z_g m / 2}{(Z_g - 2a_{fg})m / 2} \cos \alpha$$

$$= \cos^{-1} \left(\frac{125}{121} \times 0.94 \right) = \cos^{-1}(0.97107)$$

$$= 13.814^\circ$$



Problem on Gear Tooth Correction in Internal Gearing :

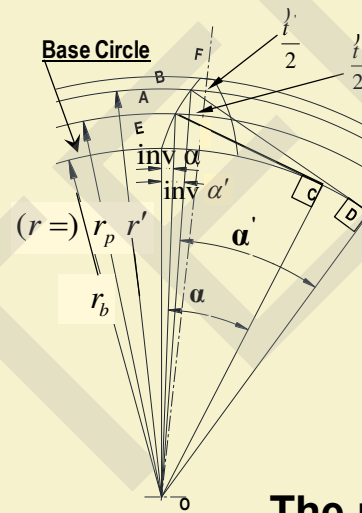
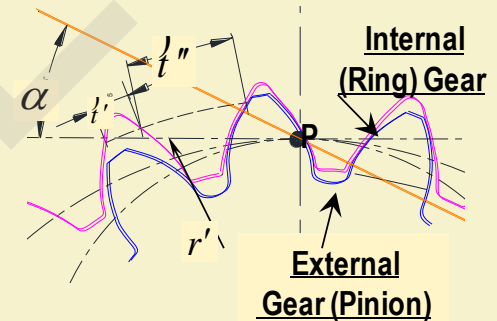
Solution (contd...) :

Ring Gear tooth tip thickness (Contd.) :

$$\begin{aligned} \text{inv } \alpha_{r'} &= \tan \alpha_{r'} - \alpha_{r'} \\ &= \tan 13.814^\circ - \frac{13.814 \times \pi}{180} \\ &= 0.24588 - 0.2411 = 0.004780 \end{aligned}$$



$$\begin{aligned} \therefore t' &= 2r' \left(\frac{t}{2r} + \text{inv } \alpha - \text{inv } \alpha' \right) \\ &= 2 \times 121 \times \left(\frac{7.854}{2 \times 125} + 0.0149 - 0.004780 \right) \\ &= 242 \times (0.031416 + 0.0149 - 0.004780) \\ &= 242 \times 0.041536 = 10.051712 \text{ mm} \end{aligned}$$



The ring gear tooth thickness:

$$\begin{aligned} t'' &= \frac{2\pi r'}{Z_g} - t' = \frac{2\pi \times 121}{50} - 10.051712 \\ &= 15.2053 - 10.051712 = 5.1536 \text{ mm} \end{aligned}$$

Problem on Gear Tooth Correction in Internal Gearing :

Solution (contd...) :

Contact Ratio:

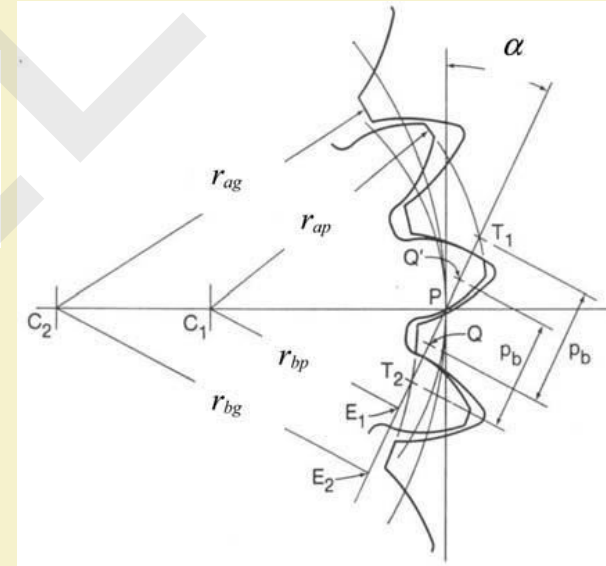
Referring to the illustration, the **Contact Ratio** of internal gearing (involute straight toothed) is expressed as:

$$\text{Contact Ratio} = \frac{T_1P + PT_2}{p_b}$$

$$\therefore C_{ri} = \frac{\sqrt{(r_{ap}^2 - r_{bp}^2)} - r_{bp} \tan \alpha + r_{bg} \tan \alpha - \sqrt{(r_{ag}^2 - r_{bg}^2)}}{\pi m \cos \alpha_o}$$

Where,

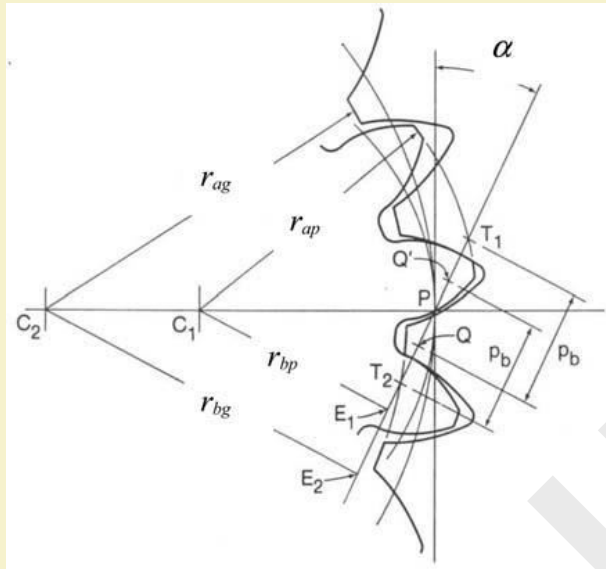
As working, α_o is standard pressure angles. They are equal in this case as there is no change in centre distance.



Problem on Gear Tooth Correction in Internal Gearing :

[Solution \(contd...\) :](#)

Contact Ratio (contd...) :



$$r_{bg} = r_{pg} \cos \alpha_o = 125 \times 0.9396 = 117.462 \text{ mm}$$

$$r_{bp} = r_{pp} \cos \alpha_o = 100 \times 0.9396 = 93.96 \text{ mm}$$

Substituting the values, the Contact Ratio :

$$C_{ri} = \frac{\left[\sqrt{(104^2 - 93.96^2)} - 93.96 \times \tan 20^\circ \right. \\ \left. + 117.465 \times \tan 20^\circ - \sqrt{(121^2 - 117.465^2)} \right]}{\pi \times 5 \times \cos 20^\circ}$$

$$C_{ri} = \frac{[(44.58 - 34.2) + (42.75 - 29.03)]}{14.76} \\ = \frac{(10.38 + 13.72)}{14.76} = 1.633$$

**The Contact ratio
is Satisfactory**

Note: Contact ratio is independent of module.

Thank You



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Module-07

Introduction to Involute Gear Tooth Correction

Lecture-38

Tooth Tip Interference, Avoidance and Contact Ratio in Involute Internal Gearing



Outline of the Lecture

- **Tip Interference in Internal Involute Gearing**
 - **Tip Interference During Engagement (Ring Gear Driver)**
 - **Tip Interference During Disengagement (Ring Gear Driver)**
 - **Avoidance of Tip Interferences in Internal Gearing by Tooth Tip Truncation (Shorter Addendum)**
 - **Avoidance of Tip Interferences in Internal Gearing by Tooth Correction**
- **Tip Interference in Harmonic Drives with Involute Toothed Gear Set**
 - **Contact Ratio Verification.**

Avoidance of 'Tip Interference' in Involute Internal Gearing :

Referring to Fig. 1 , to avoid 'Tip Interference' during engagement: $\theta_p > \left(\frac{Z_g}{Z_p} \right) \theta_g \quad \dots (1)$

Angles and θ_p are θ_g estimated as follows:

$$\theta_g = \cos^{-1} \left(\frac{r_{ag}^2 + A^2 - r_{ap}^2}{2Ar_{ag}} \right) - \beta_g \quad \dots (2)$$

$$\theta_p = \cos^{-1} \left(\frac{r_{ag}^2 - A^2 - r_{ap}^2}{2Ar_{ap}} \right) + \beta_p \quad \dots (3)$$

Where, the subscripts p and g refer to pinion and gear respectively:

And A is the centre distance between gear and pinion.

$$\text{Also, } \beta_g = \text{inv} \alpha - \text{inv} \alpha_{tg} \quad \dots (4)$$

$$\beta_p = \text{inv} \alpha_{tp} - \text{inv} \alpha \quad \dots (5)$$

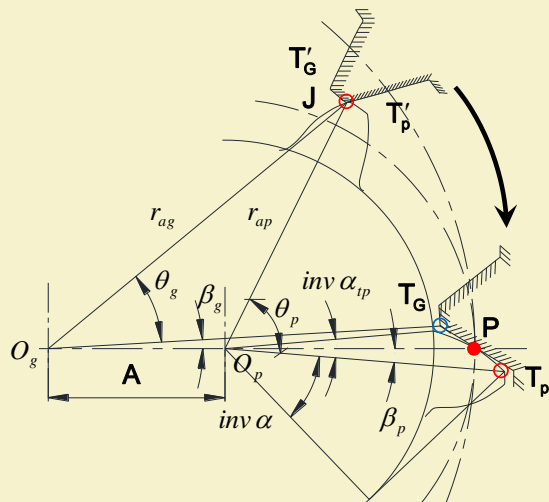


Fig.- 1 : 'Tip Interference' in Involute Internal Gearing.
(During engagement- Gear is Driver)

Avoidance of 'Tip Interference' in Involute Internal Gearing (cont...) :

Where, α_{tp} and α_{tg} , the pressure angles at pinion and gear tooth tips respectively, are derived as follows:

$$\cos \alpha_{tg} = \left(\frac{Z_g}{Z_g - 2a_{fg}} \right) \cos \alpha_o \quad \dots (6)$$

$$\cos \alpha_{tp} = \left(\frac{Z_p}{Z_p + 2a_{fp}} \right) \cos \alpha_o \quad \dots (7)$$

Where, a_{fg} and a_{fp} , are the corresponding addendum factors i.e., the ratio of addendum height (with respect to standard pitch circles) to the module for the gear and pinion respectively.

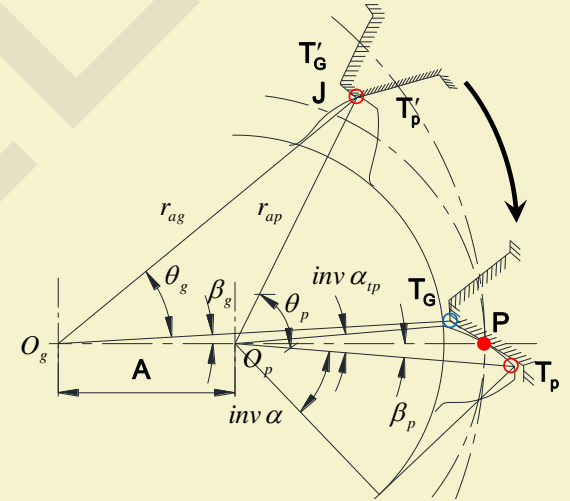


Fig.- 1 : 'Tip Interference' in Involute Internal Gearing.
(During engagement- Gear is Driver)

Avoidance of 'Tip Interference' in Involute Internal Gearing (cont...) :

Figure 2, illustrates the disengagement 'Tip Interference' of internal gearing when the ring gear is driver. The avoidance of the disengagement tip interference is to be examined as follows. Note- manufacturing tolerances in backlash free gearing plays a role.

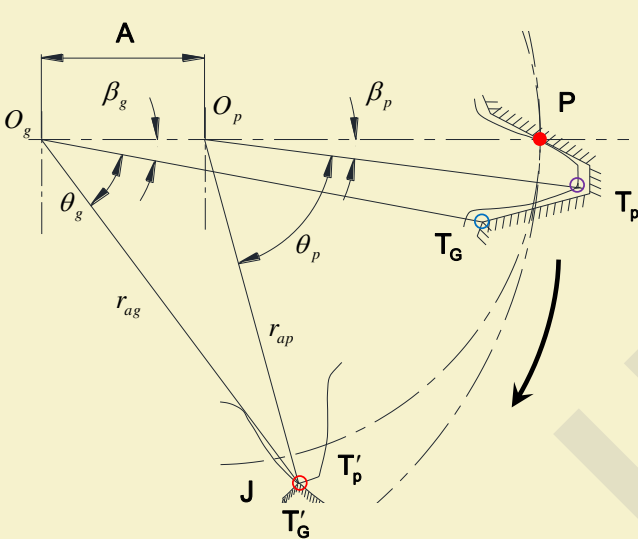


Fig.- 2 : 'Tip Interference' in Involute Internal Gearing.
(During disengagement- Gear is Driver)

$$\theta_p > \left(\frac{Z_g}{Z_p} \right) \theta_g \quad \dots (8)$$

$$\theta_g = \cos^{-1} \left(\frac{r_{ag}^2 + A^2 - r_{ap}^2}{2Ar_{ag}} \right) - \beta_g \quad \dots (9)$$

$$\theta_p = \cos^{-1} \left(\frac{r_{ag}^2 - A^2 - r_{ap}^2}{2Ar_{ap}} \right) - \beta_p \quad \dots (10)$$

$$\beta_g = \gamma_g + (\text{inv}\alpha - \text{inv}\alpha_{tg}) \quad \dots (11)$$

$$\beta_p = \gamma_p - (\text{inv}\alpha_{tp} - \text{inv}\alpha) \quad \dots (12)$$

Where, γ_g and γ_p are the angular arc thicknesses of gear tooth and pinion tooth at their respective pitch circles.

Effect of tooth modification on avoidance of 'Tip Interference' :

Tooth Truncation :

An easy solution to avoid tip interference is to truncate the addendum of either or both ring gear and pinion. However, such an action is limited by the contact ratio.

Combining profile shifting and centre distance modification is better solution keeping the contact ratio within the allowable limit.

Table -1 shows possible minimum tooth difference avoiding tip interference

i.e., $\theta_p > \left(\frac{Z_g}{Z_p}\right)\theta_g$ by tooth truncation only.

Table -1 

$(Z_g - Z_p)$	$(\alpha_o)^\circ$	Z_g	a_{fg}	a_{fp}	Contact ratio	θ_p°	θ_g°	$(Z_g/Z_p)\theta_g^\circ$
7	14.5	100	0.48	0.48	1.27	45.1	41.9	45.05
5	20		1.00	0.2	1.28	60.2	57.2	60.2
5	22.5		0.78	0.78	1.40	69.6	66.1	69.6
4	30		1.0	1.0	1.47	92.0	88.2	91.9

Effect of tooth modification on avoidance of 'Tip Interference' (cont...) :

Profile shift and CD modification :

Tooth thicknesses (arc) t_g and t_p at the respective working pitch circles are calculated as:

$$t_g = 2r_g \left(\frac{t_{go}}{2r_{go}} + \text{inv}\alpha - \text{inv}\alpha_o \right) \quad \dots (14)$$

$$t_p = 2r_p \left(\frac{t_{po}}{2r_{po}} + \text{inv}\alpha_o - \text{inv}\alpha \right) \quad \dots (15)$$

The circular pitch (arc) p_c is expressed as :

$$p_c = t_g + t_p = \frac{2\pi r_p}{Z_p} \quad \dots (16)$$

Effect of tooth modification on avoidance of 'Tip Interference' (cont...) :

Profile shift and CD modification (cont...) :

$$\text{Now } r_{go} = \frac{Z_g}{Z_p} r_{po} \quad \dots (17) \quad \text{and} \quad r_g = \frac{Z_g}{Z_p} r_p \quad \dots (18)$$

Substituting (14), (15), (17) & (18) in (16) the working pressure angle α is derived as :

$$\text{inv}\alpha = \frac{\pi m - (t_{po} + t_{go})}{m(Z_g + Z_p)} + \text{inv}\alpha_o \quad \dots (19)$$

The values of (arc) t_{go} and t_{po} at the respective standard pitch circles can be determined from the cutter geometry and amount of offset at cutting, as follows:

Effect of tooth modification on avoidance of 'Tip Interference' (cont...) :

Profile shift and CD modification (cont...) :

Considering that the pinion is cut by a rack with offset x_p , the arc tooth thickness t_{po} at standard pitch is derived as :
$$t_{po} = \frac{\pi m}{2} \pm 2x_p \tan \alpha_o \quad \dots (20)$$

Where, '+' is for withdrawal and '-' is for deepening the cutter from standard depth in tooth generation.

The centre distance A_c , between the pinion cutter and the ring gear being cut, for an offset x_g , can be expressed as:

$$A_c = (Z_g - Z_p)m / 2 + x_g = r'_g - r'_{pc} = r'_{pc} (i - 1) \quad \dots (21)$$

Where, $i = r'_g / r'_{pc} = Z_g / Z_{pc}$, in which Z_{pc} is the teeth number of pinion cutter and r'_g , r'_{pc} are working pitch circles of gear and pinion cutter respectively.

Effect of tooth modification on avoidance of 'Tip Interference' (cont...) :

Profile shift and CD modification (cont...) :

Hence ,
rearranging (21): $A_c = \frac{Z_{pc} m (i-1) \cos \alpha_o}{2 \cos \alpha_c} \dots (22)$

Therefore, $\alpha_c = \cos^{-1} \left[\frac{Z_{pc} m (i-1) \cos \alpha_o}{2 A_c} \right] \dots (23)$

Where α_c is the working pressure angle at the gear cutting condition.

After finding out the value of α_c it is substituted in (19) to calculate t'_{go} as follows:

$$t'_{go} = m[\pi + (\text{inv} \alpha_o - \text{inv} \alpha_c)(Z_g - Z_{pc})] - t'_{pc} \dots (24)$$

Where t'_{pc} is the tooth thickness of cutter at standard pitch circle and it is equal to $\pi m / 2$.

When t'_{pc} and t'_{go} are obtained α , the working pressure angle of pinion and gear in mesh at specified centre distance, is found out with the help of eqn. (19). Hence,

$$r_g = \frac{r_{go} \cos \alpha_o}{\cos \alpha} \dots (25)$$

and,

$$r_p = \frac{r_{po} \cos \alpha_o}{\cos \alpha} \dots (26)$$

Effect of tooth modification on avoidance of 'Tip Interference' (cont...) :

Profile shift and CD modification (cont...) :

The modified CD "A " of pinion and gear is: $A = r_g - r_p \quad \dots (27)$

Typical results are shown in Table – 2, below:

Table - 2

$(Z_g - Z_p)$	$(\alpha_o)^\circ$	Z_g	x_g / m	(A/A_o)	$(\alpha)^\circ$	Contact ratio
7	14.5	100	0.025	1.007	15.93	1.45
5	20		0.025	1.010	21.45	1.376
5	22.5		0.035	1.013	24.27	1.56
4	30		0.00	1.00	30.00	1.470

Comparison with the data for
Tooth truncation only.

Table -1

$(Z_g - Z_p)$	$(\alpha_o)^\circ$	Z_g	a_{fg}	a_{fp}	Contact ratio	θ_p°	θ_g°	$(Z_g/Z_p)\theta_g^\circ$
7	14.5	100	0.48	0.48	1.27	45.1	41.9	45.05
5	20		1.00	0.2	1.28	60.2	57.2	60.2
5	22.5		0.78	0.78	1.40	69.6	66.1	69.6
4	30		1.0	1.0	1.47	92.0	88.2	91.9

Avoidance of 'Tip Interference' in Harmonic Drive with Involute Internal Gearing:

Referring to Fig. 3 , to avoid 'Tip Interference' during engagement with deformed pinion:

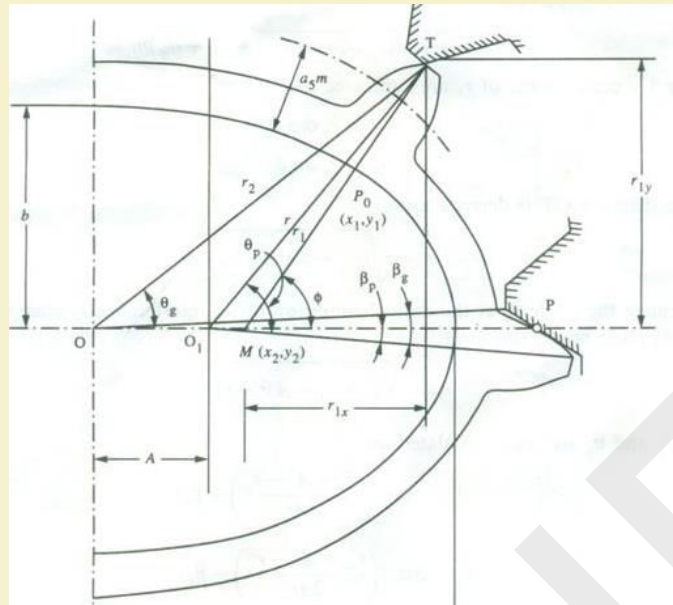


Fig.- 3 : 'Tip Interference' in Involute Internal Gearing in HD.
(During engagement- Gear is Driver)

$$\theta_p > \left(\frac{Z_g}{Z_p} \right) \theta_g \quad \dots \text{(HD_1)}$$

Angles θ_p and θ_g are estimated as follows:

$$\theta_g = \cos^{-1} \left(\frac{r_{ag}^2 + A^2 - r_{ap}^2}{2Ar_{ag}} \right) - \beta_g \quad \dots \text{(HD_2)}$$

$$\theta_p = \cos^{-1} \left(\frac{r_{ag}^2 - A^2 - r_{ap}^2}{2Ar_{ap}} \right) + \beta_p \quad \dots \text{(HD_3)}$$

Others parameters are calculated following the geometry.

It is fortunate that with standard stub teeth tip interference is avoided even with tooth difference of two in case of 20° involute gear set in Harmonic Drive.

It is due to flexsion of Flex Spline (planet Gear).

Contact Ratio (recapitulation...) :

It is essential that Contact Ratio to be verified in all cases of avoidance of tooth tip interference.

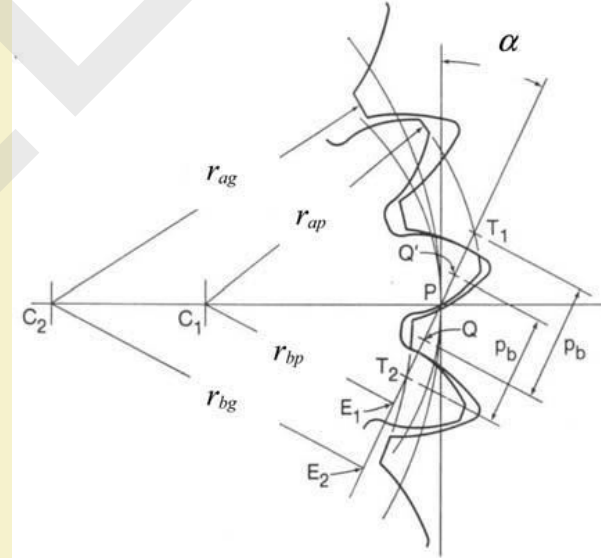
Referring to the illustration, the **Contact Ratio** of internal gearing (involute straight toothed) is expressed as:

$$\text{Contact Ratio} = \frac{T_1 P + P T_2}{P_b}$$

$$\therefore C_{ri} = \frac{\sqrt{(r_{ap}^2 - r_{bp}^2)} - r_{bp} \tan \alpha + r_{bg} \tan \alpha - \sqrt{(r_{ag}^2 - r_{bg}^2)}}{\pi m \cos \alpha_o}$$

Where,

α is working, α_o is standard pressure angles. They are equal in this case as there is no change in centre distance.



Thank You



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